MATH 449, HOMEWORK 8

DUE DECEMBER 4, 2015

Part I. Theory

Problem 1 (Süli–Mayers, Exercise 9.1)**.** Construct orthogonal polynomials of degrees 0, 1, and 2 on the interval $(0,1)$ with the weight function $w(x) = -\ln x$.

Problem 2 (Süli–Mayers, Exercise 9.2)**.** Let the polynomials φ_j, $j = 0, 1, \ldots$, form an orthogonal system on the interval $(-1,1)$ with respect to the weight function $w(x) \equiv 1$. Show that the polynomials $\varphi_j\big((2x-a-b)/(b-a)\big)$, $j = 0, 1, \ldots$, represent an orthogonal system for the interval (a,b) and the same weight function. Hence obtain the polynomials in Example 9.5 from the Legendre polynomials in Example 9.6.

Part II. Programming

Like last week, there is no sample code for this assignment, so you should create a new file `hw8.py` beginning with the usual lines:

```
from __future__ import division
from pylab import *
```

Hand in a printed copy of your code, as well as a printout of the IPython terminal session(s) containing the commands and output you used to get your answers.

Problem 3. For this problem, you will implement Gaussian quadrature for $w \equiv 1$ on the interval $(-1,1)$. Computing the roots and weights of Legendre polynomials is somewhat complicated, so rather than implement this aspect yourself, you can use the built-in SciPy function `p_roots` to do so. At the top of your source file, after the line `from pylab import *`, add the line

```
from scipy.special.orthogonal import p_roots
```

The command `[x,w] = p_roots(n+1)` will give you the quadrature points `x` and weights `w` for the degree $n+1$ Legendre polynomials on $[-1,1]$.

Create a function `gauss1(f,n)` that returns $\mathcal{G}_n(f) \approx \int_{-1}^{1} f(x)\,\mathrm{d}x$. Use this to approximate $\frac{\pi}{2} = \int_{-1}^{1} \sqrt{1-x^2}\,\mathrm{d}x$ for $n = 4, 16, 256$.

Problem 4. Using a change of variables $(-1,1) \mapsto (a,b)$ as in Problem 2, create a function `gauss(f,a,b,n)` that returns $\mathcal{G}_n(f) \approx \int_a^b f(x)\,\mathrm{d}x$. Use this to approximate $\frac{\pi}{4} = \int_0^1 \sqrt{1-x^2}\,\mathrm{d}x$ for $n = 4, 16, 256$.