MATH 456, HOMEWORK 8

DUE DECEMBER 4, 2013

Problem 1. Given S, N, and $d < R < u$, suppose we take the CRR prices $S(N,j) = Su^j d^{N-j}$ and probabilities $Q_j = Q_j' = \binom{N}{j}\pi^j(1-\pi)^{N-j}$, where $\pi = \frac{R-d}{u-d}$. Prove that the implied binomial tree computed using the "Simple as One, Two, Three" algorithm is precisely the same as the CRR tree. That is, show that $S(n,j) = Su^j d^{n-j}$ for all $n = 0, \dots, N$ and $j = 0, \dots, n$.

Hint: Use induction on N.

Problem 2. Assume that $Q_j > 0$ for $j = 0, \dots, N$ and $\sum_{j=0}^N Q_j = 1$. Prove that the probabilities $Q(n,j) = \binom{n}{j} q(n,j)$ obtained using the "Simple as One, Two, Three" algorithm also satisfy $Q(n,j) > 0$ for $j = 0, \dots, n$ and $\sum_{j=0}^n Q(n,j) = 1$ for all $n = 0, \dots, N$. (In other words, this shows that we always obtain "valid" probabilities, and we never run into the sort of problems we can encounter with implied volatility trees.)

Hint: Use induction on N, along with the identity $\binom{n}{j-1} + \binom{n}{j} = \binom{n+1}{j}$.

Problem 3. Do Exercise 12.13.

Correction: There are a few typos in this problem. First, $q(2,2)$ and $Q(2,2)$ are equal to 0.3, not $0.3333\dots$ as stated. Moreover, the formulas for $Q(2,j)$ are wrong: they should be $Q(2,0) = 1 \cdot q(2,0)$, $Q(2,1) = 2 \cdot q(2,1)$, and $Q(2,2) = 1 \cdot q(2,2)$.