2. The component functions $\cos t$, $\ln t$, and $\dfrac{1}{t-2}$ are all defined when $t > 0$ and $t \neq 2$, so the domain of $\mathbf{r}$ is $(0,2) \cup (2,\infty)$.

3. $\lim\limits_{t\to 0} e^{-3t} = e^0 = 1$, $\lim\limits_{t\to 0} \dfrac{t^2}{\sin^2 t} = \lim\limits_{t\to 0} \dfrac{1}{\dfrac{\sin^2 t}{t^2}} = \dfrac{1}{\lim\limits_{t\to 0} \dfrac{\sin^2 t}{t^2}} = \dfrac{1}{\left(\lim\limits_{t\to 0} \dfrac{\sin t}{t}\right)^2} = \dfrac{1}{1^2} = 1,$

and $\lim\limits_{t\to 0} \cos 2t = \cos 0 = 1$. Thus

$$\lim_{t\to 0}\left(e^{-3t}\,\mathbf{i} + \frac{t^2}{\sin^2 t}\,\mathbf{j} + \cos 2t\,\mathbf{k}\right) = \left[\lim_{t\to 0} e^{-3t}\right]\mathbf{i} + \left[\lim_{t\to 0} \frac{t^2}{\sin^2 t}\right]\mathbf{j} + \left[\lim_{t\to 0} \cos 2t\right]\mathbf{k} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

17. Taking $\mathbf{r}_0 = \langle 2,0,0\rangle$ and $\mathbf{r}_1 = \langle 6,2,-2\rangle$, we have from Equation 12.5.4

$\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)\,\langle 2,0,0\rangle + t\,\langle 6,2,-2\rangle,\ 0 \le t \le 1$ or $\mathbf{r}(t) = \langle 2+4t, 2t, -2t\rangle,\ 0 \le t \le 1.$

Parametric equations are $x = 2+4t,\ y = 2t,\ z = -2t,\ 0 \le t \le 1.$

32. Parametric equations for the helix are $x = \sin t$, $y = \cos t$, $z = t$. Substituting into the equation of the sphere gives $\sin^2 t + \cos^2 t + t^2 = 5 \ \Rightarrow\ 1 + t^2 = 5 \ \Rightarrow\ t = \pm 2$. Since $\mathbf{r}(2) = \langle \sin 2, \cos 2, 2\rangle$ and $\mathbf{r}(-2) = \langle \sin(-2), \cos(-2), -2\rangle$, the points of intersection are $(\sin 2, \cos 2, 2) \approx (0.909, -0.416, 2)$ and $(\sin(-2), \cos(-2), -2) \approx (-0.909, -0.416, -2)$.

41. If $t = -1$, then $x = 1,\ y = 4,\ z = 0$, so the curve passes through the point $(1,4,0)$. If $t = 3$, then $x = 9,\ y = -8,\ z = 28$, so the curve passes through the point $(9,-8,28)$. For the point $(4,7,-6)$ to be on the curve, we require $y = 1 - 3t = 7 \ \Rightarrow\ t = -2$. But then $z = 1 + (-2)^3 = -7 \neq -6$, so $(4,7,-6)$ is not on the curve.

42. The projection of the curve C of intersection onto the xy-plane is the circle $x^2 + y^2 = 4$, $z = 0$. Then we can write $x = 2\cos t,\ y = 2\sin t,\ 0 \le t \le 2\pi$. Since C also lies on the surface $z = xy$, we have $z = xy = (2\cos t)(2\sin t) = 4\cos t\sin t$, or $2\sin(2t)$. Then parametric equations for C are $x = 2\cos t,\ y = 2\sin t,\ z = 2\sin(2t),\ 0 \le t \le 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + 2\sin(2t)\,\mathbf{k},\ 0 \le t \le 2\pi$.

43. Both equations are solved for z, so we can substitute to eliminate z: $\sqrt{x^2+y^2} = 1 + y \ \Rightarrow\ x^2 + y^2 = 1 + 2y + y^2 \ \Rightarrow\ x^2 = 1 + 2y \ \Rightarrow\ y = \frac{1}{2}(x^2 - 1)$. We can form parametric equations for the curve C of intersection by choosing a parameter $x = t$, then $y = \frac{1}{2}(t^2 - 1)$ and $z = 1 + y = 1 + \frac{1}{2}(t^2 - 1) = \frac{1}{2}(t^2 + 1)$. Thus a vector function representing C is $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}(t^2 - 1)\,\mathbf{j} + \frac{1}{2}(t^2 + 1)\,\mathbf{k}$.

44. The projection of the curve C of intersection onto the xy-plane is the parabola $y = x^2$, $z = 0$. Then we can choose the parameter $x = t \ \Rightarrow\ y = t^2$. Since C also lies on the surface $z = 4x^2 + y^2$, we have $z = 4x^2 + y^2 = 4t^2 + (t^2)^2$. Then parametric equations for C are $x = t,\ y = t^2,\ z = 4t^2 + t^4$, and the corresponding vector function is $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + (4t^2 + t^4)\,\mathbf{k}$.