10. $\mathbf{r}(t) = \langle e^{-t}, t - t^3, \ln t \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle -e^{-t}, 1 - 3t^2, 1/t \rangle$

23. The vector equation for the curve is $\mathbf{r}(t) = \left\langle t^2 + 1, 4\sqrt{t}, e^{t^2 - t} \right\rangle$, so $\mathbf{r}'(t) = \left\langle 2t, 2/\sqrt{t}, (2t - 1)e^{t^2 - t} \right\rangle$. The point $(2, 4, 1)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle 2, 2, 1 \rangle$. Thus, the tangent line goes through the point $(2, 4, 1)$ and is parallel to the vector $\langle 2, 2, 1 \rangle$. Parametric equations are $x = 2 + 2t$, $y = 4 + 2t$, $z = 1 + t$.

24. The vector equation for the curve is $\mathbf{r}(t) = \langle \ln(t + 1), t \cos 2t, 2^t \rangle$, so $\mathbf{r}'(t) = \langle 1/(t + 1), \cos 2t - 2t \sin 2t, 2^t \ln 2 \rangle$. The point $(0, 0, 1)$ corresponds to $t = 0$, so the tangent vector there is $\mathbf{r}'(0) = \langle 1, 1, \ln 2 \rangle$. Thus, the tangent line goes through the point $(0, 0, 1)$ and is parallel to the vector $\langle 1, 1, \ln 2 \rangle$. Parametric equations are $x = 0 + 1 \cdot t = t$, $y = 0 + 1 \cdot t = t$, $z = 1 + (\ln 2)t$.

28. $\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, e^t \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle -2 \sin t, 2 \cos t, e^t \rangle$. The tangent line to the curve is parallel to the plane when the curve's tangent vector is orthogonal to the plane's normal vector. Thus we require $\langle -2 \sin t, 2 \cos t, e^t \rangle \cdot \langle \sqrt{3}, 1, 0 \rangle = 0 \quad \Rightarrow$
$-2\sqrt{3} \sin t + 2 \cos t + 0 = 0 \quad \Rightarrow \quad \tan t = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad t = \frac{\pi}{6}$ [since $0 \le t \le \pi$].
$\mathbf{r}\left(\frac{\pi}{6}\right) = \left\langle \sqrt{3}, 1, e^{\pi/6} \right\rangle$, so the point is $(\sqrt{3}, 1, e^{\pi/6})$.

35.
$$\begin{aligned}\int_0^2 (t\,\mathbf{i} - t^3\,\mathbf{j} + 3t^5\,\mathbf{k})\,dt &= \left(\int_0^2 t\,dt\right)\mathbf{i} - \left(\int_0^2 t^3\,dt\right)\mathbf{j} + \left(\int_0^2 3t^5\,dt\right)\mathbf{k} \\ &= \left[\tfrac{1}{2}t^2\right]_0^2\mathbf{i} - \left[\tfrac{1}{4}t^4\right]_0^2\mathbf{j} + \left[\tfrac{1}{2}t^6\right]_0^2\mathbf{k} \\ &= \tfrac{1}{2}(4 - 0)\,\mathbf{i} - \tfrac{1}{4}(16 - 0)\,\mathbf{j} + \tfrac{1}{2}(64 - 0)\,\mathbf{k} = 2\,\mathbf{i} - 4\,\mathbf{j} + 32\,\mathbf{k}\end{aligned}$$