5. $\int_1^2 \int_0^{2z} \int_0^{\ln x} xe^{-y}\,dy\,dx\,dz = \int_1^2 \int_0^{2z} \left[-xe^{-y}\right]_{y=0}^{y=\ln x} dx\,dz = \int_1^2 \int_0^{2z} \left(-xe^{-\ln x} + xe^0\right) dx\,dz$

$$= \int_1^2 \int_0^{2z} (-1 + x)\,dx\,dz = \int_1^2 \left[-x + \tfrac{1}{2}x^2\right]_{x=0}^{x=2z} dz$$

$$= \int_1^2 \left(-2z + 2z^2\right) dz = \left[-z^2 + \tfrac{2}{3}z^3\right]_1^2 = -4 + \tfrac{16}{3} + 1 - \tfrac{2}{3} = \tfrac{5}{3}$$

9. $\iiint_E y\,dV = \int_0^3 \int_0^x \int_{x-y}^{x+y} y\,dz\,dy\,dx = \int_0^3 \int_0^x \left[yz\right]_{z=x-y}^{z=x+y} dy\,dx = \int_0^3 \int_0^x 2y^2\,dy\,dx$

$$= \int_0^3 \left[\tfrac{2}{3}y^3\right]_{y=0}^{y=x} dx = \int_0^3 \tfrac{2}{3}x^3\,dx = \tfrac{1}{6}x^4\Big]_0^3 = \tfrac{81}{6} = \tfrac{27}{2}$$

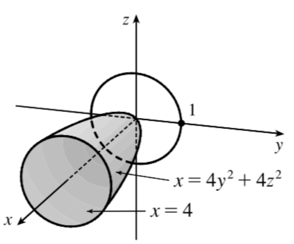
13. Here $E = \{(x, y, z) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}, 0 \le z \le 1 + x + y\}$, so

$$\iiint_E 6xy\,dV = \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy\,dz\,dy\,dx = \int_0^1 \int_0^{\sqrt{x}} \left[6xyz\right]_{z=0}^{z=1+x+y} dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} 6xy(1 + x + y)\,dy\,dx = \int_0^1 \left[3xy^2 + 3x^2y^2 + 2xy^3\right]_{y=0}^{y=\sqrt{x}} dx$$

$$= \int_0^1 (3x^2 + 3x^3 + 2x^{5/2})\,dx = \left[x^3 + \tfrac{3}{4}x^4 + \tfrac{4}{7}x^{7/2}\right]_0^1 = \tfrac{65}{28}$$

17.



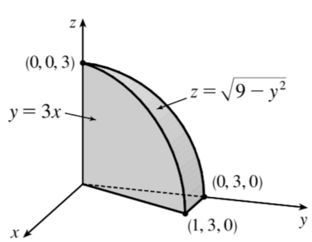
The projection of E onto the yz-plane is the disk $y^2 + z^2 \le 1$. Using polar coordinates $y = r\cos\theta$ and $z = r\sin\theta$, we get

$$\iiint_E x\,dV = \iint_D \left[\int_{4y^2+4z^2}^4 x\,dx\right] dA = \tfrac{1}{2}\iint_D \left[4^2 - (4y^2 + 4z^2)^2\right] dA$$

$$= 8\int_0^{2\pi}\int_0^1 (1 - r^4)\,r\,dr\,d\theta = 8\int_0^{2\pi} d\theta \int_0^1 (r - r^5)\,dr$$

$$= 8(2\pi)\left[\tfrac{1}{2}r^2 - \tfrac{1}{6}r^6\right]_0^1 = \tfrac{16\pi}{3}$$

18.



$$\int_0^1 \int_{3x}^3 \int_0^{\sqrt{9-y^2}} z\,dz\,dy\,dx = \int_0^1 \int_{3x}^3 \tfrac{1}{2}(9 - y^2)\,dy\,dx$$

$$= \int_0^1 \left[\tfrac{9}{2}y - \tfrac{1}{6}y^3\right]_{y=3x}^{y=3} dx$$

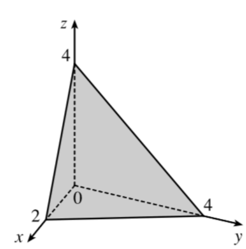
$$= \int_0^1 \left[9 - \tfrac{27}{2}x + \tfrac{9}{2}x^3\right] dx$$

$$= \left[9x - \tfrac{27}{4}x^2 + \tfrac{9}{8}x^4\right]_0^1 = \tfrac{27}{8}$$

19. The plane $2x + y + z = 4$ intersects the xy-plane when

$2x + y + 0 = 4 \quad\Rightarrow\quad y = 4 - 2x$, so

$E = \{(x, y, z) \mid 0 \le x \le 2, 0 \le y \le 4 - 2x, 0 \le z \le 4 - 2x - y\}$ and



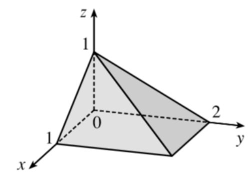
$$V = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\,dy\,dx = \int_0^2 \int_0^{4-2x} (4 - 2x - y)\,dy\,dx$$

$$= \int_0^2 \left[4y - 2xy - \tfrac{1}{2}y^2\right]_{y=0}^{y=4-2x} dx$$

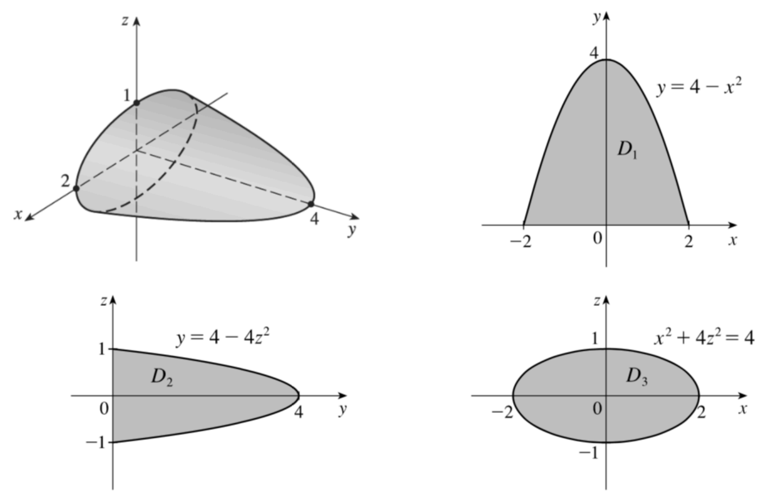
$$= \int_0^2 \left[4(4 - 2x) - 2x(4 - 2x) - \tfrac{1}{2}(4 - 2x)^2\right] dx$$

$$= \int_0^2 (2x^2 - 8x + 8)\,dx = \left[\tfrac{2}{3}x^3 - 4x^2 + 8x\right]_0^2 = \tfrac{16}{3}$$

27. $E = \{(x, y, z) \mid 0 \le x \le 1, 0 \le z \le 1 - x, 0 \le y \le 2 - 2z\}$, the solid bounded by the three coordinate planes and the planes $z = 1 - x$, $y = 2 - 2z$.



29.



If D_1, D_2, D_3 are the projections of E on the xy-, yz-, and xz-planes, then

$$D_1 = \left\{(x, y) \mid -2 \le x \le 2, 0 \le y \le 4 - x^2\right\} = \left\{(x, y) \mid 0 \le y \le 4,\ -\sqrt{4 - y} \le x \le \sqrt{4 - y}\right\}$$

$$D_2 = \left\{(y, z) \mid 0 \le y \le 4,\ -\tfrac{1}{2}\sqrt{4 - y} \le z \le \tfrac{1}{2}\sqrt{4 - y}\right\} = \left\{(y, z) \mid -1 \le z \le 1, 0 \le y \le 4 - 4z^2\right\}$$

$$D_3 = \left\{(x, z) \mid x^2 + 4z^2 \le 4\right\}$$

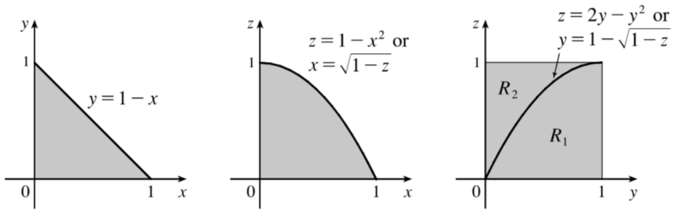
Therefore

$$\begin{aligned}
E &= \left\{(x, y, z) \mid -2 \le x \le 2, 0 \le y \le 4 - x^2,\ -\tfrac{1}{2}\sqrt{4 - x^2 - y} \le z \le \tfrac{1}{2}\sqrt{4 - x^2 - y}\right\} \\
&= \left\{(x, y, z) \mid 0 \le y \le 4,\ -\sqrt{4 - y} \le x \le \sqrt{4 - y},\ -\tfrac{1}{2}\sqrt{4 - x^2 - y} \le z \le \tfrac{1}{2}\sqrt{4 - x^2 - y}\right\} \\
&= \left\{(x, y, z) \mid -1 \le z \le 1, 0 \le y \le 4 - 4z^2,\ -\sqrt{4 - y - 4z^2} \le x \le \sqrt{4 - y - 4z^2}\right\} \\
&= \left\{(x, y, z) \mid 0 \le y \le 4,\ -\tfrac{1}{2}\sqrt{4 - y} \le z \le \tfrac{1}{2}\sqrt{4 - y},\ -\sqrt{4 - y - 4z^2} \le x \le \sqrt{4 - y - 4z^2}\right\} \\
&= \left\{(x, y, z) \mid -2 \le x \le 2,\ -\tfrac{1}{2}\sqrt{4 - x^2} \le z \le \tfrac{1}{2}\sqrt{4 - x^2}, 0 \le y \le 4 - x^2 - 4z^2\right\} \\
&= \left\{(x, y, z) \mid -1 \le z \le 1,\ -\sqrt{4 - 4z^2} \le x \le \sqrt{4 - 4z^2}, 0 \le y \le 4 - x^2 - 4z^2\right\}
\end{aligned}$$

Then

$$\begin{aligned}
\iiint_E f(x, y, z)\, dV &= \int_{-2}^{2} \int_0^{4-x^2} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z)\, dz\, dy\, dx = \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z)\, dz\, dx\, dy \\
&= \int_{-1}^{1} \int_0^{4-4z^2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z)\, dx\, dy\, dz = \int_0^4 \int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z)\, dx\, dz\, dy \\
&= \int_{-2}^{2} \int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2} \int_0^{4-x^2-4z^2} f(x, y, z)\, dy\, dz\, dx = \int_{-1}^{1} \int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}} \int_0^{4-x^2-4z^2} f(x, y, z)\, dy\, dx\, dz
\end{aligned}$$

34.



The projections of E onto the xy- and xz-planes are as in the first two diagrams and so

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\,dy\,dz\,dx = \int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x,y,z)\,dy\,dx\,dz$$
$$= \int_0^1 \int_0^{1-y} \int_0^{1-x^2} f(x,y,z)\,dz\,dx\,dy = \int_0^1 \int_0^{1-x} \int_0^{1-x^2} f(x,y,z)\,dz\,dy\,dx$$

Now the surface $z = 1 - x^2$ intersects the plane $y = 1 - x$ in a curve whose projection in the yz-plane is $z = 1 - (1 - y)^2$ or $z = 2y - y^2$. So we must split up the projection of E on the yz-plane into two regions as in the third diagram. For (y, z) in R_1, $0 \le x \le 1 - y$ and for (y, z) in R_2, $0 \le x \le \sqrt{1 - z}$, and so the given integral is also equal to

$$\int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dy\,dz + \int_0^1 \int_{1-\sqrt{1-z}}^1 \int_0^{1-y} f(x,y,z)\,dx\,dy\,dz$$
$$= \int_0^1 \int_0^{2y-y^2} \int_0^{1-y} f(x,y,z)\,dx\,dz\,dy + \int_0^1 \int_{2y-y^2}^1 \int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dz\,dy.$$