

Math 310 Homework 2, Due Sept. 20, 2007

(1) Prove that for every natural number $m \neq 1$, there is one and only one natural number q such that $m = q'$. (10 points)

(2) Prove:

(i): $mn = nm$ for all natural numbers m and n . (10 points)

(ii): $(mn)p = m(np)$ for all natural numbers m, n , and p . (10 points)

(3) Prove that for every natural number n , no natural number m exists such that $n < m$ and $m < n + 1$. (10 points)

(4) Let \mathbb{N} be the set of natural numbers and let $S = \mathbb{N} \times \mathbb{N}$. Define an equivalence relation \sim on S by setting $(x, y) \sim (u, v)$ when $xv(y + u) = yu(x + v)$.

(i): Show that \sim is an equivalence relation. (10 points)

(ii): List ten elements of the equivalence set that $(1, 1)$ belongs to. (5 points)