(1) Prove that for any real numbers $x$ and $\alpha$ with $\alpha > 1$, there is an integer $n$ such that $x < n < x + \alpha$. (10 points)

(Hint: It suffices to consider the case when $x \geq 0$ (WHY?). Let $S$ be the set of all natural numbers greater or equal to $x + \alpha$. $S$ is not empty (WHY?). Consider the smallest element in $S$, whose existence is warranted by the well-ordering principle.)

(2) Prove that for any real numbers $x < y$, there is a rational number $r$ such that $x < r < y$. (10 points)

(3) Assume $\lim_{n \to \infty} a_n = a$ and $\lim_{n \to \infty} b_n = b$. Prove the following

(a) $\lim_{n \to \infty} (a_n + b_n) = a + b$. (10 points)
(b) $\lim_{n \to \infty} a_n b_n = ab$. (10 points)
(c) If $b_n \neq 0$ for all $n$ and $b \neq 0$, then $\lim_{n \to \infty} a_n/b_n = a/b$. (10 points)