

**Math 310 Homework 5, Fall 2007, Due Dec. 4**

(1) Let  $a_1 = \sqrt{2}$ ,  $a_2 = \sqrt{2 + \sqrt{2}}$ ,  $a_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$ ,  $\dots$ . In general,  $a_{n+1} = \sqrt{2 + a_n}$ .

(a) Show that  $\lim_{n \rightarrow \infty} a_n$  exists. (10 points)

(Hint: Show that the sequence is increasing and bounded.)

(b) Find the exact limit of the sequence  $(a_n)$ . (5 points)

(2a) Show by the limit properties in Problem 3, Homework 4, that

$$\lim_{n \rightarrow \infty} \frac{n^3}{n^3 - n^2 - n - 1} = 1.$$

(5 points)

(2b) Show directly by definition (the  $\epsilon - N$  format) that the limit in

(2a) is 1. (10 points)

(3) Let  $p(x) = x^2 - 10x + 1$ .

(a) Show by the method in (2a) that if  $\lim_{n \rightarrow \infty} a_n = 1$ , then  $\lim_{n \rightarrow \infty} p(a_n) = -8$ . (5 points)

(b) Show by the method in (2b) that the limit is  $-8$ . (10 points)