Math 310 Midterm, Fall 2007, Due Oct. 30

(1a) Use the mathematical induction to show that for all \( n \in \mathbb{N} \),
\[
1 + 2 + 3 + \cdots + n = n(n + 1)/2.
\]
(5 points)

(1b) Use \((n + 1)^2 - n^2 = 2n + 1\) and (1a) to show
\[
1 + 2^2 + 3^2 + \cdots n^2 = n(n + 1)(2n + 1)/6
\]
(5 points)

(1c) Find the formula for
\[
1^3 + 2^3 + 3^3 + \cdots n^3.
\]
(5 points)

(2) Show that if \( p, q \) and \( r \) are natural numbers such that \( p < r \) and \( q < r \), then \( |p - q| < r \). (5 points)

(3a) For any two natural numbers \( m \) and \( n \), show that there is a natural number \( q \) such that \( mq > n \). (10 points)

(3b) Conclude that for any two positive rational numbers \( r \) and \( s \), there is a natural number \( n \) such that \( nr > s \). (5 points)

(4) Prove that \( n!/n^n \) is a null sequence. (10 points)
   (Hint: \( k/n \leq 1 \) when \( k \leq n \) and \( k/n \leq 1/2 \) when \( k \leq n/2 \).)

(5) Show that the definition of the product of two real numbers is well-defined independent of the representatives chosen. (10 points)