

**Math 310 Midterm, Fall 2007, Due Oct. 30**

(1a) Use the mathematical induction to show that for all  $n \in \mathbb{N}$ ,

$$1 + 2 + 3 + \cdots + n = n(n + 1)/2. \quad (5 \text{ points})$$

(1b) Use  $(n + 1)^2 - n^2 = 2n + 1$  and (1a) to show

$$1 + 2^2 + 3^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6 \quad (5 \text{ points})$$

(1c) Find the formula for

$$1 + 2^3 + 3^3 + \cdots + n^3. \quad (5 \text{ points})$$

(2) Show that if  $p, q$  and  $r$  are natural numbers such that  $p < r$  and  $q < r$ , then  $|p - q| < r$ . (5 points)

(3a) For any two natural numbers  $m$  and  $n$ , show that there is a natural number  $q$  such that  $mq > n$ . (10 points)

(3b) Conclude that for any two positive rational numbers  $r$  and  $s$ , there is a natural number  $n$  such that  $nr > s$ . (5 points)

(4) Prove that  $n!/n^n$  is a null sequence. (10 points)

(Hint:  $k/n \leq 1$  when  $k \leq n$  and  $k/n \leq 1/2$  when  $k \leq n/2$ .)

(5) Show that the definition of the product of two real numbers is well-defined independent of the representatives chosen. (10 points)