Math 450 - Homework 1 - Solutions
(Exercises from Lecture Notes 1)

1. Exercise 2.2

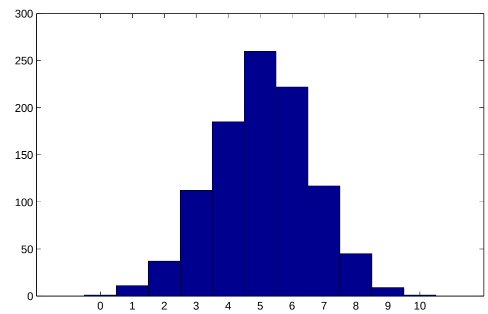

Figure 1: Histogram for 1000 runs of the experiment: toss a coin 10 times and count the number of heads.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
u=121; %seed for the multiplicative congruence algorithm
K=7^5;
M=2^31-1;

a=[];
for j=1:1000
    for i=1:10
        u=rem(K*u,M);
        x=u/M;
        a(i,j)=(x<=1/2);
    end
end
y=sum(a);
hist(y,0:10)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

2. Exercise 2.3

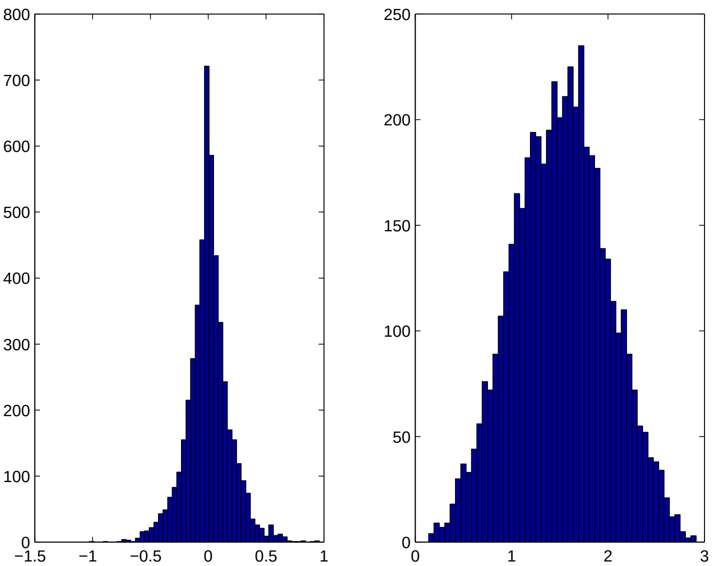

Figure 2: Histograms with 50 bins for the determinant (left) and the trace (right) of 5000 3-by-3 random matrices with entries uniformly distributed on $[0, 1]$. The trace, as one might expect, is more concentrated on $3/2$. The determninant seems to be sharply concentrated around 0.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
d=[];
t=[];
for i=1:5000
    A=rand(3,3);
    d=[d det(A)];
    t=[t trace(A)];
end
subplot(1,2,1); hist(d,50)
subplot(1,2,2); hist(t,50)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

3. Exercise 2.4

   (a) Let the random variable Z represent the output of `rand`. The command `X=2*Z` simulates a random variable with the uniform distribution on $[0, 2]$. If $X \in [0, 1)$ then `floor(X)` outputs 0, and if $X \in [1, 2)$, floor(X) outputs 1. Therefore, floor(2*Z) yields 0 with probability $P(Z \in [0, 1/2)) = 1/2$ and 1 with probability $P(Z \in [1/2, 1)) = 1/2$.

(b) Now, round(X) gives 0 if $X \in [0, 1/2)$, 1 if $X \in [1/2, 3/2)$, and 2 if $X \in [3/2, 2)$. Therefore, round(2*Z) yields 0 with probability $P(Z \in [0, 1/4)) = 1/4$, 1 with probability $P(Z \in [1/4, 3/4)) = 1/2$ and 2 with probability $P(Z \in [3/4, 1)) = 1/4$.

4. Exercise 2.6

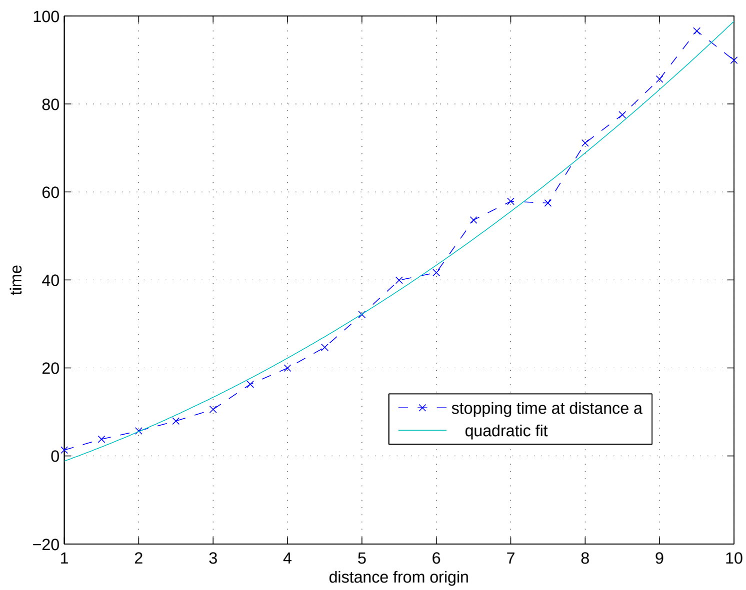

Figure 3: Graph of the stopping time at distance a from the origin, using simulation of the random walk on the plane defined in Lecture Notes 1, section 1.3.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
r=1;
u=19;
t=[];
k=10;
for a=1:0.5:k
    S=[];
    for j=1:100
        Z=[0 0];
        for n=1:1000000000000
```

```
            s=ceil(u*rand);
            Z=Z+r*[cos(2*pi*s/u) sin(2*pi*s/u)];
            if Z(1)^2+Z(2)^2>=a^2
                S=[S n];
                break
            end
        end
    end
    t=[t sum(S)/100];
end
a=1:0.5:k;
plot(a,t)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

5. Exercise 2.8

   We can use our `samplefrommarkov` program to simulate a long run, say 10000 long, of the Markov chain.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
X=samplefrommarkov([0.5 0.5], [0.75 0.25;0.25 0.75], 10000);
r=sum(X==1)/10000
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

   The above gave me the value $r = 0.4951$ for the frequency of rainy days. (Your number should be close but will certainly be different.) The matrix P in this case has eigenvalues 0.5 and 1. The eigenvector associated to 1, normalized, is $(0.5, 0.5)$. We will see later that the components of this vector are the long range relative frequencies of rainy and fair whether. The relative frequency obtained by simulation is approximating 0.5. We will understand the relationship later when we study Markov chains systematically.