

Math 450 - Homework 5

Due date: Friday, 2/23/07

1. Read sections 1.5, 1.6 and 1.7 of the textbook. (Also review sections 1.3 and 1.4, which are needed for problems 2, 3, and 4.)
2. Do Exercise 1.3.2 of the textbook. In addition to answering the questions posed by the problem, write down the stochastic matrix of the Markov chain, draw the corresponding digraph indicating the states, the transition probabilities, and the absorbing states. Confirm your answer with a computer simulation by running the Markov chain 1000 times and counting the fraction of times the gambler wins \$10. (Such a program was used to solve the cat and mouse problem of HW3.)
3. Go back to the cat and mouse problem of HW 3 and show the probability that the mouse reaches the wall hole before visiting the cat is 0.6. (Do this by solving the appropriate linear system numerically.)
4. Let X_0, X_1, X_2, \dots be a Markov chain with state space S . Decide whether the random variable T in each case below is a stopping time or not. Explain your answer. (The definition of *stopping time* is on page 19, section 1.4, of textbook.)
 - (a) T is the time of the r th visit to state $i \in S$, where r is a positive integer.
 - (b) $T = T_A + n_0$, where A is a subset of S , T_A is the hitting time at A , and n_0 is a positive integer.
 - (c) $T = T_A - n_0$, where A is a subset of S , T_A is the hitting time at A , and n_0 is a positive integer.
 - (d) T is the first nonnegative integer n such that $X_{n+1} = 1$, and $T = \infty$ if $X_{n+1} \neq i$ for all $n + 1$.
5. Do Exercise 1.5.1 of the textbook.
6. Do Exercise 1.5.2 of the textbook.
7. Do Exercise 1.6.1 of the textbook.