Math 450 - Homework 7

Due date: Friday, 3/09/07

1. Read sections 1.8 and 1.9 (pages 40 to 52) of textbook.
2. (Exercise 1.8.2, page 46 of textbook.) Find the invariant distribution of the transition matrix in Exercise 1.1.7:

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \\ p & 1-p & 0 \end{pmatrix}.$$

3. Exercise 1.8.4, page 46 of textbook. (Also draw a transition diagram and write the matrix P.) The algebra for calculating the stationary distribution π in this problem is a bit tedious. If you wish, simply take the values $\alpha_1 = 1/2, \alpha_2 = 1/3, \alpha_3 = 1/6$ and solve the appropriate system of equations numerically. But if you persist in working out the general case, you will find the analytic solution rather pleasing.
4. Exercise 1.9.1 (a), (b), (c), page 51 of textbook.
5. (Book shuffling) Do a simulation of the situation described in exercise 1.8.4. More precisely, assume that each morning the student takes one of $n = 3$ books from his shelf, each with equal probabilities, independently of the previous day's choice. In the evening he replaces the book at the left-hand end of the shelf. We want to find how often the shelf returns to the initial state. In your simulation, assume that the books are initially ordered as $1, 2, \ldots, n$ from left to right. Each move consists of a permutation of the form

$$(1, 2, \ldots, i-1, i, i+1, \ldots, n) \mapsto (i, 1, 2, \ldots, i-1, i+1, \ldots, n).$$

Repeat the operation 100000 times. (This took about 9 seconds on my laptop.)