Math 450 - Homework 9

Due date: Friday, 3/30/07

1. Read chapter 3, sections 1 through 6 (pages 108-123).
2. Do exercise 3.3.1, page 114.
3. Do exercise 3.6.2, page 123.
4. Perform the following simulated experiment. A number of balls, some red and some black, are distributed between two urns. The balls are numbered from 1 to N. At random times one person picks a number between 1 and N and transfers the corresponding ball from its urn to the other. Independent of the first person and also at random times, a second person picks a number between 1 and N and replaces the corresponding ball with a red one if the chosen ball is from urn I, or black if the chosen ball is from urn II, keeping the new ball in the same urn as the old one. (Note that if, for example, the chosen ball is red and it was in urn I, then the action does not change anything.) We assume that the sequence of actions of the first person have independent holding times which are exponential of parameter q_1, and the actions of the second person have independent holding times which are exponential of parameter q_2. We wish to find the long term fraction, b, of the N balls that are black and lie in urn II. Do this for the values $q_1 = 1$ and $q_2 = 0.1, 2$, and 20. For each value of q_2, draw a graph of the fraction of balls that are black and in urn II as a function of time. I suggest taking the following parameters: total number of balls: 100, total number of events (actions of the two persons): 20000. (It took about 45 seconds for each of the three runs of 20000 events.) What are the approximate values b for each q_2? Give a qualitative explanation for the values you obtain.