Math 450 - Homework 11

Due date: Friday, 4/13/07

1. Read section 4.4, pages 159-165. Also read sections 1 and 2 of D.J. Higham’s paper, pages 525-529.

2. We fix a time $T$ and wish to observe the distribution of positions of $W_T$, the one-dimensional Brownian motion. Do this as follows. Using the program `bpath2.m` in Higham’s paper, simulate a large number of sample paths of $W_t$ for $0 \leq t \leq T$ and plot the empirical distribution of values of $W_T$. (Suggestion: simulate 10000 sample paths and calculate the fraction of outcomes over 50 equally spaced bins. This is easily done using the `hist` command. On the same coordinate system plot the graph of the theoretical values that would be obtained under the exact normal distribution with mean 0 and variance $T$. (Suggestion: use a style of plot similar to that of figure 4 on page 15 of lecture notes set 3.) Do this in two separate graphs, one for $T = 0.1$ and another for $T = 1.0$.

3. Adapt the program `bpath2.m` to obtain sample paths of a (discretized) two-dimensional Brownian path. (Suggestion: use time interval $[0, 1]$ and $N = 10000$ time steps. For a better looking graph, use the `axis equal` command so that the $x$- and $y$-coordinate axis display the same length scale.)

4. Obtain a plot similar to that of figure 2, page 529 of Higham’s paper, but now for the process 

$$X_t = \cos(t + W_t).$$

(The graph of figure 2 corresponds to the process $X_t = \exp(t + \frac{1}{2}W_t)$.)