# Math 450 - Homework 11 

Due date: Friday, 4/13/07

1. Read section 4.4, pages 159-165. Also read sections 1 and 2 of D.J. Higham's pager, pages 525-529.
2. We fix a time $T$ and wish to observe the distribution of positions of $W_{T}$, the one-dimensional Brownian motion. Do this as follows. Using the program bpath2.m in Higham's paper, simulate a large number of sample paths of $W_{t}$ for $0 \leq t \leq T$ and plot the empirical distribution of values of $W_{T}$. (Suggestion: simulate 10000 sample paths and calculate the fraction of outcomes over 50 equally spaced bins. This is easily done using the hist command. On the same coordinate system plot the graph of the theoretical values that would be obtained under the exact normal distribution with mean 0 and variance $T$. (Suggestion: use a style of plot similar to that of figure 4 on page 15 of lecture notes set 3.) Do this in two separate graphs, one for $T=0.1$ and another for $T=1.0$.
3. Adapt the program bpath2.m to obtain sample paths of a (discretized) two-dimensional Brownian path. (Suggestion: use time interval $[0,1]$ and $N=10000$ time steps. For a better looking graph, use the axis equal command so that the $x$ - and $y$-coordinate axis display the same length scale.)
4. Obtain a plot similar to that of figure 2, page 529 of Higham's paper, but now for the process

$$
X_{t}=\cos \left(t+W_{t}\right)
$$

(The graph of figure 2 corresponds to the process $X_{t}=\exp \left(t+\frac{1}{2} W_{t}\right)$.)

