iPhone dropped from top of bulding 1000 m high

$s =$ how far it has fallen $(m)$ after $t$ seconds $= 4.9t^2 = f(t)$

Q1: When does the iPhone hit the ground? Which of the following is the best estimate (don't use a calculator!)

A) after approximately 6 sec

B) after approximately 10 sec

C) after approximately 14 sec

D) after approximately 20 sec

E) I don't know

(What is the exact answer?)

Representation (partial) of $s = f(t) = 4.9t^2$ using a table

| $\boldsymbol{t}$ | $\boldsymbol{s = 4.9t^2 = f(t)}$ |
|---|---|
| (sec) | (meters) |
| | |
| 0 | 0 |
| 1 | 4.9 |
| 1.5 | 11.025 |
| 1.9 | 17.6890 |
| 1.99 | 19.4045 |
| 1.999 | 19.5804 |
| **2** | **19.6000** |
| 2.001 | 19.6196 |
| 2.01 | 19.7965 |
| 2.1 | 21.6090 |
| 2.5 | 30.6250 |
| 3 | 44.1000 |
| 4 | 78.4000 |

Average velocity between times $t = 1$ and $t = 2$ :

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{f(2) - f(1)}{2 - 1} = \frac{19.6 - 4.9}{2 - 1} = \frac{14.7}{1} = 14.7000 \text{ m/sec}$$

Average velocity between times $t = 2$ and $t = 3$ :

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{f(3) - f(2)}{3 - 2} = ???$$

Average velocity between time $t = 2$ and some other time $t = a$ :

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{f(a) - f(2)}{a - 2} = \frac{4.9(a^2) - 4.9(2^2)}{a - 2} = ???$$

Q2: Average velocity between time $t = 2$ and some other time $t = a$ simplifies to

A) $4.9a + 2$ B) $4.9a + 4.9$

C) $4.9a + 9.8$ D) $4.9a + 4.9$

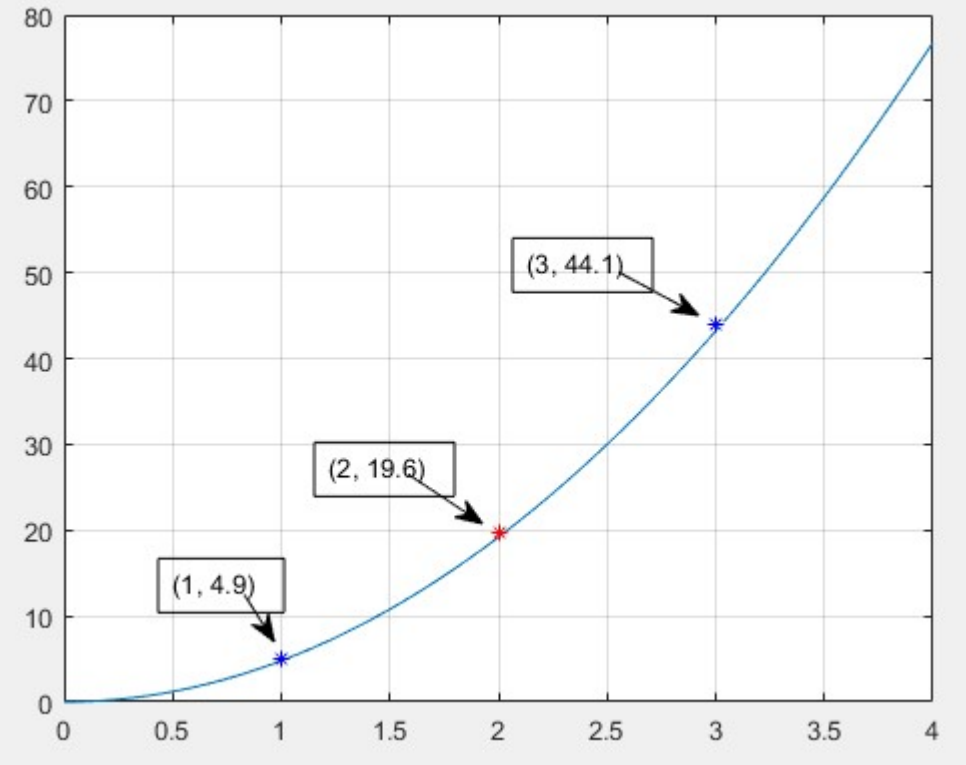
E) I don't know

Here's a table of some calculated average velocities when $s = 4.9t^2$

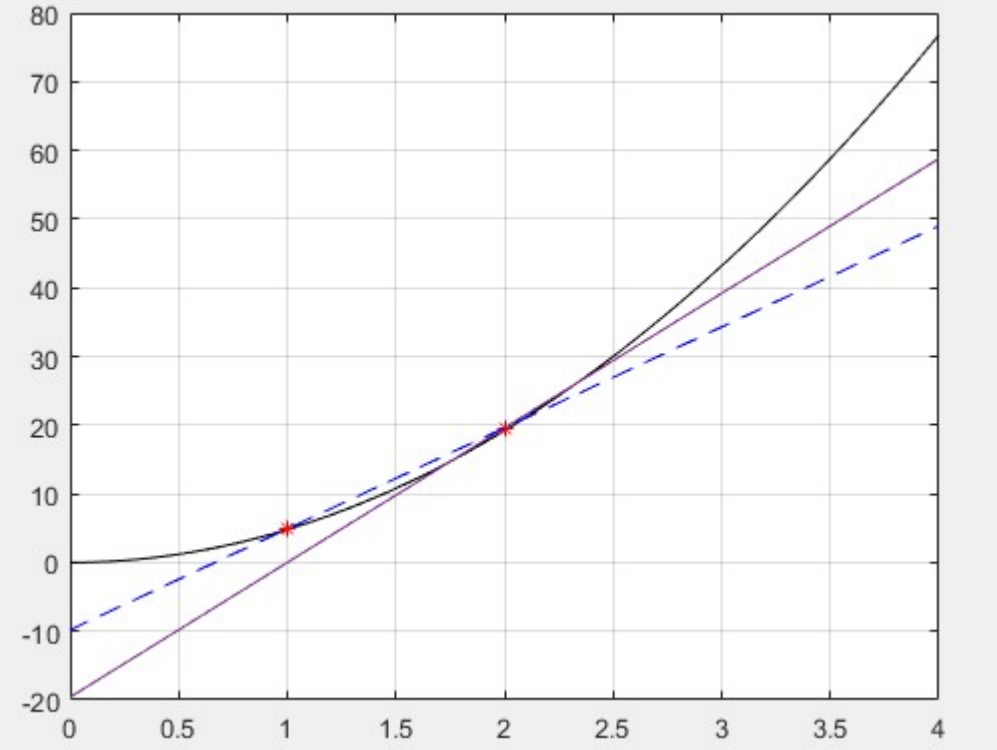
$$\text{average velocity} = \frac{\text{change in distance}}{\text{change in time}}$$

| time interval (sec) | average velocity (m/sec) | calculation of average velocity |
|---|---|---|
| $1 \le t \le 2$ | 14.7000 | $= \frac{f(2)-f(1)}{2-1} = \frac{19.6-4.9}{2-1}$ |
| $1.5 \le t \le 2$ | 17.1500 | |
| $1.9 \le t \le 2$ | 19.1100 | |
| $1.99 \le t \le 2$ | 19.5510 | |
| $1.999 \le t \le 2$ | 19.5951 | $= \frac{f(2)-f(1.999)}{2-1.999} = \frac{19.6-19.5804}{0.001}$ |
| | | |
| **"$2 \le t \le 2$"** | **???? ---> ????** | **???? $= \frac{f(2)-f(2)}{2-2} = \frac{0}{0}$ ??????** |
| | | |
| $2 \le t \le 2.001$ | 19.6049 | $= \frac{f(2.001)-f(2)}{2.001-2} = \frac{19.6-4.9}{0.001}$ |
| $2 \le t \le 2.01$ | 19.6490 | |
| $2 \le t \le 2.1$ | 20.0900 | |
| $2 \le t \le 2.5$ | 22.0500 | |
| $2 \le t \le 3$ | 24.5000 | |

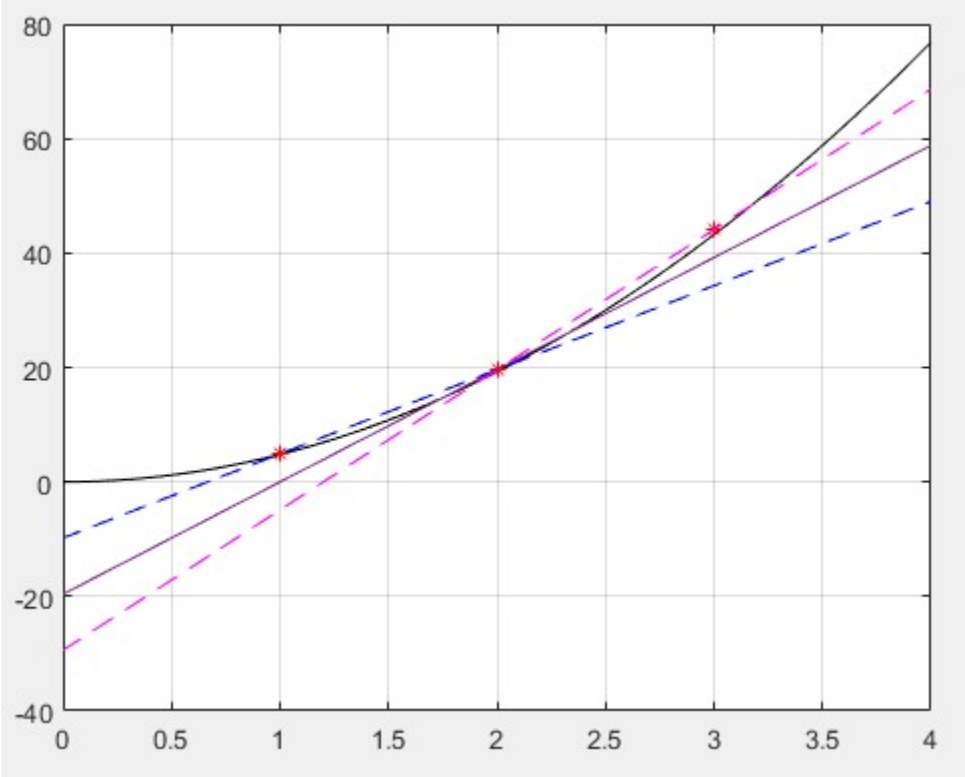
Graph of $s = f(t) = 4.9t^2$ (partial: only for $0 \leq t \leq 4$, not for the entire fall)



The average velocity for $1 \leq t \leq 2 \ = \frac{f(2)-f(1)}{2-1}$. On the graph of $s = f(t)$, this fraction represents what geometric quantity?



What geometric quantity does $\frac{f(3)-f(2)}{3-2}$ represent ?




What geometric quantity does $\frac{f(a)-f(2)}{a-2}$ represent ?

SUMMARY:

For $s = f(t) =$ the distance fallen by time $t$, the quantity $\frac{f(a) - f(2)}{a - 2}$ can be thought of as either

$\begin{cases} \text{average velocity for between times } a \text{ and } 2 \\ \text{the slope of the "secant" line through } (2, f(2)) \text{ and } (a, f( \end{cases}$

The <u>limit</u> of $\frac{f(a) - f(2)}{a - 2}$

as $a$ gets closer and closer to $2 = 19.6$ can be thought of as

either

$\begin{cases} \text{the } \underline{\text{instantaneous velocity}} \text{ of the falling iPhone at the inst} \\ \text{the slope of the } \underline{\text{tangemt}} \text{ line to the graph at } (2, f(2)). \end{cases}$

http://www.sfu.ca/~jtmulhol/calculus-applets/html/appletsforcalculus.html