Indeterminate Forms

"$\frac{0}{0}$" or "$\frac{\pm\infty}{\pm\infty}$"

L'Hopital's Rule applies to that kind of indeterminate form:

L'Hôpital's Rule:

Suppose

$f(x)$ and $g(x)$ are differentiable near $x = a$ (not necessarily at $x = a$)
$g'(x) \neq 0$ near $x = a$
$\lim_{x \to a} \frac{f(x)}{g(x)}$ is one of the indeterminate forms "$\frac{0}{0}$" or "$\frac{\pm\infty}{\pm\infty}$".

Then

If $\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = L$ also.

Notes: 1) L'Hôpital's Rule works if "$x \to a$" is replaced by "$x \to a^+$" or "$x \to a^-$"
2) In L'Hôpital's Rule, it's OK if either a or L is $\pm\infty$.

More examples of indeterminate forms

L'Hopital's Rule does not apply directly to the following forms; but usually the limit can be manipulated into a form where L'Hopital's Rule does apply.

"$0 \cdot \infty$":: all of the following have this form, but each has a different limit

$\lim_{x\to\infty} (2x)(\frac{1}{x})$ $\lim_{x\to\infty} (13x)(\frac{1}{x})$ $\lim_{x\to\infty} (2x^2)(\frac{1}{x})$ $\lim_{x\to\infty} (2x)(\frac{1}{x^2})$

"$\infty - \infty$": all of the following have this form, but each has a different limit

$\lim_{x\to\infty} (x+7) - x$ $\lim_{x\to\infty} x - (x+13)$ $\lim_{x\to\infty} x^2 - x$ $\lim_{x\to\infty} x - x^2$

"1^∞": all of the following have this form, but each has a different limit

$\lim_{x\to 0^+} (3^x)^{1/x}$ $\lim_{x\to 0^+} (\pi^x)^{1/x}$ $\lim_{x\to\infty} (1+\frac{1}{x})^x$ $\lim_{x\to\infty} (1+\frac{a}{x})^{bx}$

"∞^0": all of the following have this form, but each has a different limit

$\lim_{x\to 0^+} (\frac{1}{x})^x$ $\lim_{x\to\infty} (x+e^x)^{\frac{1}{x}}$

"0^0": all of the following have this form, but each has a different limit

$\lim_{x\to 0^+} x^x$ $\lim_{x\to 0^+} x^{\,e/(1+\ln x)}$

Examples of forms that are <u>not indeterminate</u>:

"$\infty \cdot \infty$" " $\frac{\infty}{0}$ " " $\frac{0}{\infty}$ " " 0^∞ " "∞^∞"