Math 131, Fall 2016
Quiz 3, October 6, 2016
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
<u>Do NOT use any methods except those discussed so far in this course.</u>

1. The point $(0, 3)$ is on the graph of $y = \sqrt{3x^2 - 6x + 9}$. What is the slope of the line <u>perpendicular to</u> the tangent line at $(0, 0)$?

$$\frac{dy}{dx} = \frac{1}{2\sqrt{3x^2-6x+9}} \cdot (6x-6)$$

at $x=0$, derivative $= \frac{1}{2\sqrt{9}}(-6) = \frac{-6}{6} = -1$ (slope of tan. line at $(0,0)$)

$\underbrace{\text{The line perpendicular to the tan line at } (0,0)}_{(= \text{ normal line at } (0,0))}$ therefore has slope 1

2. $f(x) = \frac{a\cos x}{x+e^x}$ where a is a constant. If $f'(0) = 7$, what is a?

Quotient Rule:

$$f'(x) = \frac{(x+e^x)(a+\cos x)' - (a\cos x)(x+e^x)'}{(x+e^x)^2}$$

$$= \frac{(x+e^x)(-\sin x) - (a\cos x)(1+e^x)}{(x+e^x)^2}$$

So $f'(0) = \frac{(1)(0) - (a)(2)}{1^2} = 7$

so $-2a = 7$

$a = -7/2$

Math 131, Fall 2016
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For all 9 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.

1. Let $y = \tan(x^3 - 3x)$. What is the equation of the line perpendicular to the tangent line at the point where $x = 0$?

$$\frac{dy}{dx} = \sec^2(x^3-3x)\cdot\frac{d}{dx}(x^3-3x)$$
$$= \sec^2(x^3-3x)\cdot(3x^2-3)$$

at $x=0$, $\frac{dy}{dx} = \sec^2(0)\cdot(-3) = -3$ (slope of tan. line where $x=0$)

so the equation of the line $\perp$ to the tan line where $x=0$ is $(y-0) = \frac{1}{3}(x-0)$ or $y = \frac{1}{3}x$.

$\tan(0^3-3\cdot 0)$

2. $f(x) = \frac{\cos x}{b + \sin x}$ where b is a positive constant. The slope of the tangent line to the graph at $(0, \frac{1}{b})$ is $-\frac{1}{4}$. What is b?

$$f'(x) = \frac{(b+\sin x)(\cos x)' - (\cos x)(b+\sin x)'}{(b+\sin x)^2} = \frac{-\sin x\cdot(b+\sin x)-(\cos x)(\cos x)}{(b+\sin x)^2}$$

$$\text{so } f'(0) = \frac{-(0)(b)-(1)(1)}{(b+0)^2} = \frac{-1}{b^2} = -\frac{1}{4}$$

so $b^2 = 4$

Since b is positive, $b = 2$

Math 131, Fall 2016
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For all 10 a.m. Sections
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1. If $h(x) = 10^{3x^2+6x+2}$, what is $h'(0)$?

$$h'(x) = (\ln 10) \cdot 10^{3x^2+6x+2} \cdot \frac{d}{dx}(3x^2+6x+2)$$

$$= \ln 10 \cdot 10^{3x^2+6x+2} (6x+6)$$

so $h'(0) = (\ln 10) \cdot 10^2 \cdot 6 = 600 \ln 10$

$f'(0)=4$

2. Suppose $f(0) = 3$ and the tangent line to the graph of f at $(0, 3)$ is $y = 4x + 3$

$g(0) = 6$ and the tangent line to the graph of g at $(0, 6)$ is $y = -2x + 6$

$g'(0) = -2$

What is $(\frac{f}{g})'(0)$?

Quotient Rule

$$\left(\frac{f}{g}\right)'(0) = \frac{g(0) f'(0) - f(0)(g'(0))}{(g(0))^2}$$

$$= \frac{6(4) - (3)(-2)}{6^2} = \frac{30}{36} = \frac{5}{6}$$

Math 131, Fall 2016
Quiz 3, October 6, 2016
For all 11 a.m. Sections
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Do NOT use any methods except those discussed so far in this course.

1. What is the slope of the tangent line to the graph of $y = e^{x^2+(4/x^2)}$ at the point $(1, e^5)$?

$$\frac{dy}{dx} = e^{x^2+\frac{4}{x^2}} = \frac{d}{dx} e^{x^2+4x^{-2}} = e^{x^2+\frac{4}{x^2}} \cdot \left(2x - 8x^{-3}\right)$$

$$= e^{x^2+4/x^2}\left(2x - 8/x^3\right).$$

at $x=1$, the derivative $= e^5 \cdot \left(2 - \frac{8}{1}\right) = -6e^5$

slope of tan line at $(1, e^5)$

2. Suppose $h(x) = f(x)g(x)$ and that

$h'(0) = 6$

$f(0) = 2$, and the tangent line to $y = f(x)$ at $(0, 2)$ is $y = 3x + 2$

$f'(0)=3$

$g'(0) = 4$, and $(0, b)$ is on the graph of $g(x)$

What is b?

$$h'(0) = f(0)g'(0) + f'(0)g(0)$$

$$6 = (2)(4) + (3)\underbrace{g(0)}_{= b}$$

so $3b = -2$

$b = -2/3$

Math 131, Fall 2016

Quiz 3, October 6, 2016
For all 12 p.m. Sections
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Do NOT use any methods except those discussed so far in this course.

1. Suppose $h(x) = \sin(g(x))$, where $g(x) = \pi(x + \frac{1}{2})$. What is the slope of the tangent line to $y = h(x)$ at the point $(\frac{1}{2}, 0)$?

$$h'(x) = \cos(g(x)) \cdot g'(x)$$

$$g(x) = \pi\left(x+\tfrac{1}{2}\right) = \pi x + \tfrac{\pi}{2}$$
$$g'(x) = \pi \quad (\text{constant})$$
$$g\left(\tfrac{1}{2}\right) = \pi\left(\tfrac{1}{2}+\tfrac{1}{2}\right) = \pi$$

$$h'\left(\tfrac{1}{2}\right) = \cos\left(g\left(\tfrac{1}{2}\right)\right) \cdot \pi$$
$$= \cos(\pi) \cdot \pi$$
$$= -\pi$$

2. Suppose that $h(x) = \frac{1}{g(x)}$, $h'(2) = \frac{1}{6}$, and $g'(2) = -1$.

If $g(2)$ must be a positive number, what is $g(2)$?

Quot. Rule:

$$h'(x) = \frac{-g'(x)}{(g(x))^2}$$

$$\frac{1}{6} = h'(2) = \frac{-(-1)}{(g(2))^2} = \frac{1}{(g(2))^2}$$

So $(g(2))^2 = 6$

since $g(2)$ must be positive, $g(2) = \sqrt{6}$