Math 131, Fall 2016
Quiz 5, October 20, 2016
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
<u>Do NOT use any methods except those discussed so far in this course.</u>
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$.

1. On the planet Krypton, at time $t = 0$, a rock is thrown down from a cliff that is 320m high. Its height after t seconds is $s = -8t^2 - 48t + 320$. What is the velocity of the rock when it hits the ground (the "impact velocity")? *Include the units with your answer.*

Rock hits the ground when $s = -8(t^2 + 6t - 40)$
$= -8(t+10)(t-4) = 0$ so $\cancel{t = -10}$
$t = 4$

$v = -16t - 48$
at $t = 4$.
$v(4) = -16(4) - 48 = -112$ (m/sec)

2. A population of bacteria of size 200 grows at a constant relative rate. The population has grown to 1000 after 3 weeks. How long did it take for the population to double in size? *Include the units with your answer.*

$P_0 = 200$
$P = 200e^{kt}$,
the population has doubled when $400 = 200e^{kt}$
$2 = e^{kt}$
$\ln 2 = kt$
$t = \ln 2 / k$.

But what is k?
Know $1000 = 200e^{k \cdot 3}$
$5 = e^{3k}$
$\ln 5 = 3k$
$k = (\ln 5)/3$

So pop. has doubled when $t = \frac{\ln 2}{k} = \frac{\ln 2}{\ln 5/3} = \frac{3 \ln 2}{\ln 5}$ (weeks)

Math 131, Fall 2016
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For all 9 a.m. Sections
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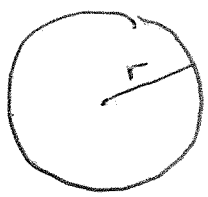
1. A rock is thrown into the middle of a lake creating a circular ripple that travels outward at a speed of 60 cm/sec. When $t = 1$ sec, how fast is the area inside the circle increasing? *Include the units with your answer.*



$\frac{dr}{dt} = 60$ when $t=1$, $r = 60$

Area $A = \pi r^2$

so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

when $t=1$, $\frac{dA}{dt} = 2\pi(60)(60) = 7200\pi$ (cm²/sec)

2. A radioactive isotope decays at a constant relative rate and has a half life of 2 years. If you begin with 8g of the isotope, how long will it take until only 3g remains? *Include the units with your answer.*

$m_0 = 8$ $\quad m = m_0 e^{kt} = 8e^{kt}$

Half life = 2 yrs, so $4 = 8e^{k \cdot 2}$

$\frac{1}{2} = e^{2k}$

$2k = \ln(\frac{1}{2})$

$k = \frac{1}{2}\ln(\frac{1}{2})$ (or $-\frac{\ln 2}{2}$)

We want the t for which

$3 = 8e^{kt}$

$\frac{3}{8} = e^{kt}$

$\ln(3/8) = kt$

$\frac{\ln(3/8)}{k} = t$

So $t = \frac{\ln(3/8)}{-\frac{\ln 2}{2}} = -\frac{2\ln(3/8)}{\ln 2}$ (yrs)

Math 131, Fall 2016
Quiz 5, October 20, 2016
For all 10 a.m. Sections
Show enough work to make it clear how you got your answer.
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1. A cylinder is growing in such a way that its height and radius are always equal. The volume of the cylinder is growing at a constant rate of 100 cm^3/min. How fast is the height changing when the radius is 10 cm? *Include the units with your answer.*

Since $r = h$

$$V = \pi r^2 h = \pi h^3$$

$$\frac{dV}{dt} = 3\pi h^2 \frac{dh}{dt}$$

when $r = 10$, Then $h = 10$

so $100 = 3\pi(10^2)\frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{100}{(3\pi)(10^2)} = \frac{1}{3\pi} \quad (\text{cm/min})$$

2. A population of size 200 grows at a constant relative rate to size 400 in 3 weeks. At what time t is the population 600? *Include the units with your answer.*

$P_0 = 200$

$$P = 200e^{kt}$$

in 3 weeks: $400 = 200e^{kt} = 200e^{3k}$

$$2 = e^{3k}$$

$$\ln 2 = 3k$$

so $k = \frac{\ln 2}{3}$

we want t for which

$$600 = 200e^{kt}$$

$$3 = e^{kt}$$

$$\ln 3 = kt$$

so $t = \frac{\ln 3}{k} = \frac{\ln 3}{\frac{\ln 2}{3}} = \frac{3\ln 3}{2}$ (wks)

Math 131, Fall 2016
Quiz 5, October 20, 2016
For all 11 a.m. Sections
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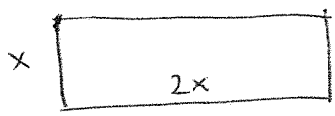
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$.

1. The sides of a rectangle are growing. The longer side is always twice aw's long as the shorter side, and the area is growing at a rate of 10 cm^2/sec. When the shorter side is 4cm long, how fast is the longer side growing? *Include the units with your answer.*



$A = 2x^2$

$\frac{dA}{dt} = 4x \frac{dx}{dt} = 10$

when $x = 4$

$4(4) \frac{dx}{dt} = 10 \longrightarrow \frac{dx}{dt} = \frac{10}{16} = \frac{5}{8}$ (cm/sec)

Note $\frac{5}{8} = \frac{dx}{dt}$

The longer side has length $2x$

so $\frac{d}{dx}(2x) = 2\frac{dx}{dt} = 2(5/8) = \frac{10}{8} = \frac{5}{4}$ (cm/sec)

2. A radioactive isotope decays at a constant relative rate. An initial quantity of 4g decays to 3g after 1 week. What is the half life of the isotope ? *Include the units with your answer.*

$m_0 = 4$

$m = 4e^{kt}$

For $t=1$: $3 = 4e^{k \cdot 1}$

$\frac{3}{4} = e^k$

$\ln(3/4) = k$

We want the t for which $2 = 4e^{kt}$

$\frac{1}{2} = e^{kt}$

$\ln(1/2) = kt$

$t = \frac{\ln(1/2)}{\ln(3/4)}$ $\left(\text{or } = -\frac{\ln 2}{\ln(3/4)}\right)$ (wks)

Math 131, Fall 2016
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For all 12 p.m. Sections
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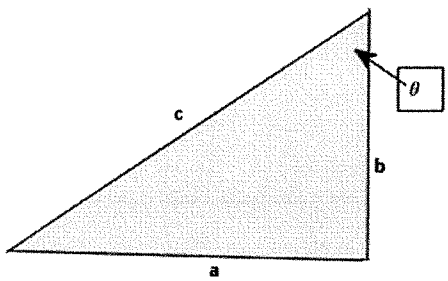
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$.

1. The figure shows a right triangle with hypotenuse c and legs of length a and b. At the moment when $a = b = 1$cm, a is growing at a rate of 1 cm/sec and b is shortening at a rate of 1 cm/sec. How fast is the angle θ (between sides b and c) changing at that moment? *Include the units with your answer.*



$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$ so $a = b\tan\theta$

$\frac{da}{dt} = b\sec^2\theta\frac{d\theta}{dt} + \frac{db}{dt}\tan\theta$

when $a = b = 1$: $\frac{da}{dt} = 1$, $\frac{db}{dt} = -1$

$\tan\theta = 1 \rightsquigarrow \theta = \frac{\pi}{4} \rightsquigarrow \sec\theta = \sqrt{2}$,
so $\sec^2\theta = 2$

Therefore when $a = b = 1$:

$1 = (1)(2)\frac{d\theta}{dt} + (-1)(1)$

$2 = 2\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = 1$ (rad/sec)

2. A population is dying off at a constant relative rate. The population on January 1, 2016 is 1000, and by January 1, 2017, it is reduced to size 400. What will be the size of the population on January 1, 2018 ? *Include the units with your answer.*

wombats

$t=0$: Jan 1, 2016: $P_0 = 1000$
$P = 1000e^{kt}$

$t=1$: Jan 1, 2017: $400 = 1000e^{k\cdot 1}$

$\frac{2}{5} = e^k$ so $k = \ln(2/5)$

$t=2$: Jan 1, 2018: $P = 1000e^{2\ln(2/5)}$ (wombats)

not necessary for credit

[which could simplify:
$P = 1000e^{2\ln(2/5)}$
$= 1000e^{\ln(2/5)^2} = 1000e^{\ln 4/25}$
$= \frac{4}{25}\cdot 1000 = 160$ wombats]