Math 131, Fall 2016
Quiz 6, November 3, 2016
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6\ln 2}$.

1. What is the absolute maximum value of $f(x) = xe^{-x}$ on the interval $[0, 100]$ and where does the absolute maximum occur?

Since f(x) is a continuous function on a closed interval [0,100] we know that an abs. max. value exists.

$f'(x) = x(-e^{-x}) + (1)(e^{-x}) = e^{-x}(-x+1) = 0$ when $x = 1$

The abs max must occur at one of

endpts $\begin{cases} 0 \\ 1 \\ 100 \end{cases}$. Test the values: $f(0) = 0$, $f(1) = e^{-1}$, $f(100) = 100e^{-100} = \frac{100}{e^{100}} \approx 0$

Abs. Max value is e^{-1} occurring at $x = 1$

2. Suppose $s = \sqrt{t^2+7}$ Use differentials to estimate the change Δs when t changes from $t = 3$ to 2.99.

$$\Delta s \approx ds = \frac{2t}{2\sqrt{t^2+7}}\,dt = \frac{t}{\sqrt{t^2+7}}\,dt$$

$$\text{so } ds = \frac{3}{\sqrt{16}}(-.01) = (-.01)\left(\frac{3}{4}\right)$$

$$(= -0.0075)$$

Math 131, Fall 2016
Quiz 6, November 3, 2016
For all 9 a.m. Sections
Show enough work to make it clear how you got your answer.
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Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6\ln 2}$.

1. Suppose f is continuous on $[6, 15]$ and differentiable on $(6, 15)$. If $f(6) = -2$ and $f'(x) \leq 10$ for all x in $(6, 16)$, what is the largest possible value for $f(15)$?

By the Mean Value theorem:

$$f(15) - f(6) = f'(z)(15-6)$$
$$= 9f'(z) \text{ for some } z \text{ in } (6,15)$$
$$f(15) = f(6) + 9f'(z)$$
$$\leq -2 + 9(10) = 88$$

2. What is the absolute minimum value of $f(x) = \frac{\ln x}{x}$ on the interval $[1, e^2]$ and where does the absolute maximum occur?

Since $f(x)$ is a continuous function on a closed interval $[1, e^2]$ we know that an absolute min value exists.

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \ln x\,(1)}{x^2} = \frac{1-\ln x}{x^2} = 0$$
$\hookrightarrow$ when $\ln x = 1$, $x = e$

The absolute minimum must occur at one of the following points:

endpoints $\to x = 1$
$x = e$
endpoints $\to x = e^2$

Test the values:

$f(1) = \frac{\ln 1}{1} = 0$ $\leftarrow$

$f(e) = \frac{\ln(e)}{e} = \frac{1}{e}$

$f(e^2) = \frac{\ln(e^2)}{e^2} = \frac{2}{e^2}$

So the absolute minimum value is 0, occurring at $x = 1$

Math 131, Fall 2016
Quiz 6, November 3, 2016
For all 10 a.m. Sections
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Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$.

1. Let $f(x) = x^3 + 6x^2 + 6x$ on $[-6, 0]$. The Mean Value Theorem guarantees that there will be one (or more) z that have a certain property. Find all these z values.

$$\frac{f(b)-f(a)}{b-a} = f'(z) \text{ for some } z \text{ in } (-6,0)$$

$$\frac{f(0)-f(-6)}{0-(-6)} = \frac{0-(-36)}{0-(-6)} = 6 = 3z^2+12z+6$$

so
$3z^2+12z=0$
$3z(z+4)=0$
so $z=-4$ (since z must be in $(-6,0)$)

2. What is the absolute maximum value of $f(x) = x - 1 + \frac{4}{x+1}$ on the interval $[0, 3]$ and where does the absolute maximum occur.

Since f is a <u>continuous</u> function on the <u>closed</u> interval $[0,3]$ we know that an absolute max value exists.

$$f'(x) = 1 - \frac{4}{(x+1)^2} = \frac{(x+1)^2-4}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2} = \frac{(x+3)(x-1)}{(x+1)^2} = 0$$

when $x = \cancel{-3}, 1$

not in domain of f.

The absolute max value must occur at one of the points:

endpoints $\to x=0$
$x=1$
endpoints $\to x=3$

Test the values:

$f(0) = -1+4=3$
$f(1) = \frac{4}{2}=2$
$f(3) = 2+\frac{4}{4}=3$

abs max value $=3$ and it occurs at $x=0, x=3$

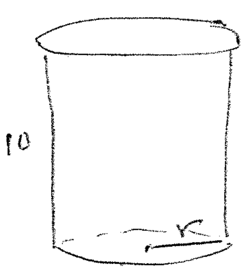
Math 131, Fall 2016
Quiz 6, November 3, 2016
For all 11 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$.

1. Suppose $s = f(t)$ is the position (m) at time t (sec) of a point moving along a line for times $0 \le t \le 10$. The point starts at $s = -3$ and ends at $s = 16$. The Mean Value Theorem tells you that at some time t the velocity must equal a certain number. What is that number? (*Include units with the answer.*)

velocity at some time $z = f'(z) = \frac{f(10) - f(0)}{10 - 0} = \frac{16-(-3)}{10}$

$= 1.9$ m/sec

2. A cylinder has height $h = 10$ m and radius $r = 5$ m. Use differentials to estimate the change in the volume of the cylinder if the radius decreases to 4.98. (*Include units with the answer.*)



$V = \pi r^2 h = 10\pi r^2$

$\Delta V \approx dV = 20\pi r\,dr$

$= 20\pi(5)(-.02)$

$= (100)(-.02)\pi$

$= -2\pi$ m^3

Math 131, Fall 2016
Quiz 6, November 3, 2016
For all 12 p.m. Sections
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Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$.

1. Suppose $s = f(t)$ is the position (m) at time t (sec) of a point moving left/right along the x-axis for times $0 \le t \le 10$. The point starts at $s = -3$ $(t = 0)$.
If you know that $-4 \le f'(t) \le 7$ at all times, then what is the largest possible value for the ending position $f(10)$? (*Include units with the answer.*)

By The Mean Value Theorem:

$$f(10) - f(0) = f'(z)(10-0)$$
$$f(10) = f(0) + f'(z)(10-0)$$
$$\le -3 + 10(7) = 67 \quad (m)$$

2. What is the absolute maximum value of $f(x) = xe^x$ on the interval $[-2, 0]$ and where does the absolute maximum occur?

Since $f(x)$ is a continuous function on a closed interval $[-2,0]$ we know that an absolute maximum value exists.

$f'(x) = x(e^x) + (1)(e^x) = e^x(x+1) = 0$ when $x = -1$

The absolute maximum value must occur at one of the following points:

endpoints $\to$ $x = -2$
$x = -1$
endpoints $\to$ $x = 0$

Test the values:
$f(-2) = -2e^{-2}$
$f(-1) = -e^{-1}$
$f(0) = 0 \leftarrow$

absolute max value is 0, and it occurs at $x = 0$.