Math 131, Fall 2016
Quiz 8, December 1, 2016
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$ *or* e^{-1}.

1. If the product of two positive numbers is 25, what is the smallest possible value for their sum?

$xy = 25$ so $y = 25/x$

Sum $= S = x + y = x + 25/x \quad (x > 0)$

$S' = 1 - \frac{25}{x^2} = 0 \rightarrow \frac{25}{x^2} = 1 \rightarrow x = 5$ (since $x > 0$)

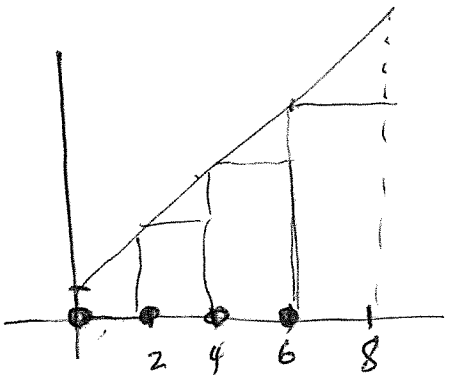
Since $S' = \frac{x^2 - 25}{x^2}$ is $\begin{cases} > 0 \text{ when } x > 5 \\ < 0 \text{ when } 0 < x < 5 \end{cases}$

$x = 5$ gives an absolute minimum value for S.

$x = 5 \rightarrow y = 5$

so smallest possible $S = x + 5 = 5 + 5 = 10$

2. For $y = f(x) = 2x + 1$, what is the "left end-point approximation" L_4 to the area under the graph and over the interval $[0, 8]$? Does L_4 overestimate or underestimate the exact area A? why?



$\Delta x = \frac{8 - 0}{4} = 2$

$L_4 = \Delta x \left(f(0) + f(2) + f(4) + f(6) \right)$

$= 2 (1 + 5 + 9 + 13) = 56$

Since f is increasing, L_4 underestimates A.

Math 131, Fall 2016
Quiz 8, December 1, 2016
For all 9 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$ *or* e^{-1}.

1. A rock is dropped off a cliff 800m high on the planet Nefron. It hits the ground 4 seconds later. What is the acceleration due to gravity on Nefron? Include units with your answer. (*Assume that no other forces such as air resistance act on the falling rock.*)

accel = a

$v = at + C$. $v(0)=0 \to C=0$

$v = at$

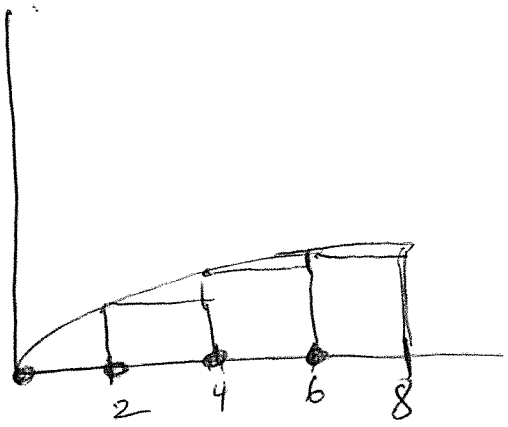
$s = \frac{1}{2}at^2 + D$ $s(0)=850 \to D=800$

$s = \frac{1}{2}at^2 + 800$

$s=0$ when $t=4$ so: $0 = \frac{1}{2}a(16)+800 \to 8a = -800$

$a = -100 \text{ m/sec}^2$

2. 2. For $y = f(x) = \sqrt{x}$, what is the "left end-point approximation" L_4 to the area under the graph and over the interval $[0, 8]$? Does L_4 overestimate or underestimate the exact area A? why?



$\Delta x = \frac{8-0}{4} = 2$

$L_4 = \Delta x \left(f(0) + f(2) + f(4) + f(6) \right)$

$= 2\left(0 + \sqrt{2} + 2 + \sqrt{6}\right)$

$= 4 + 2\sqrt{2} + 2\sqrt{6}$

Since f is increasing, L_4 underestimates A.

Math 131, Fall 2016
Quiz 8, December 1, 2016
For all 10 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$ *or* e^{-1}.

1. If the sum of two positive numbers is 25, what is the largest possible value for their product?

$x + y = 25$ so $y = 25 - x$

Product $P = xy = x(25-x) = 25x - x^2 \quad (x>0)$

$P' = 25 - 2x = 0$

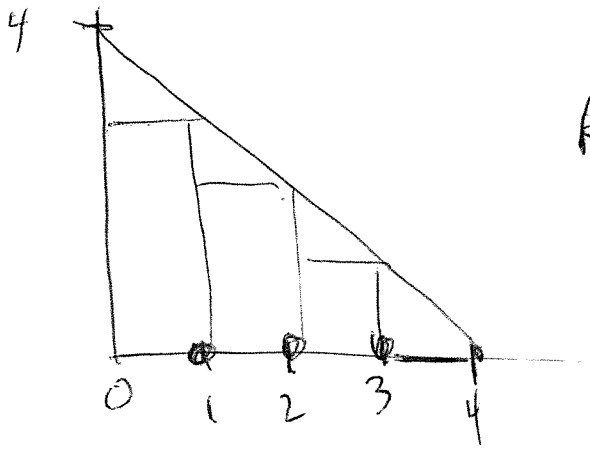
$2x = 25 \rightarrow x = \frac{25}{2}$

Since $P'(x)$ is $\begin{cases} <0 \text{ if } x > 25/2 \\ >0 \text{ if } 0 < x < \frac{25}{2} \end{cases}$

$x = \frac{25}{2}$ gives an abs. max value for P

$y = 25 - x = 25 - \frac{25}{2} = \frac{25}{2}$, so largest $P = \left(\frac{25}{2}\right)\left(\frac{25}{2}\right) = \frac{625}{4}$

2. For $y = f(x) = 4 - x$, what is the "right end-point approximation" R_4 to the area under the graph and over the interval $[0, 4]$? Does R_4 overestimate or underestimate the exact area A? why?



$\Delta x = \frac{4-0}{4} = 1$

$R_4 = \Delta x \left(f(1) + f(2) + f(3) + f(4) \right)$

$= 1 \left(3 + 2 + 1 + 0 \right)$

$= 6$

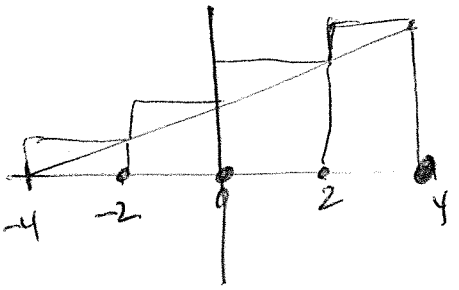
Since f is decreasing, R_4 underestimates A.

Math 131, Fall 2016
Quiz 8, December 1, 2016
For all 11 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$ *or* e^{-1}.

1. On the planet Nefron, the acceleration due to gravity is -100m/sec^2. If a stone is dropped off a cliff that is 800m high, it will hit the ground with what velocity? Include units with your answer. (*Assume that no other forces such as air resistance act on the falling rock.*)

$a = -100$
$v = -100t + C$ $\quad v(0) = 0 \rightarrow C = 0$
$v = -100t$
$s = -50t^2 + D$ $\quad s(0) = 800 \rightarrow D = 800$
$s = -50t^2 + 800$
Hits the ground when $s = 0 = -50t^2 + 800$
$50t^2 = 800$
$t^2 = \frac{800}{50} = 16$
$t = 4$ (since $t > 0$)
when $t = 4$,
$v = -100(4) = -400$ m/sec.

2. For $y = f(x) = x + 4$, what is the "right end-point approximation" R_4 to the area under the graph and over the interval $[-4, 4]$? Does R_4 overestimate or underestimate the exact area A? why?



$\Delta x = \frac{4 - (-4)}{4} = 2$

$R_4 = \Delta x \left(f(2) + f(0) + f(2) + f(4) \right)$
$= 2 \left(2 + 4 + 6 + 8 \right)$
$= 2(20) = 40$

since f is increasing, R_4 overestimates A

Math 131, Fall 2016
Quiz 8, December 1, 2016
For all 12 p.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$ *or* e^{-1}.

1. If the product of two positive numbers is 4, what is the smallest possible value for the sum of the squares of the numbers?

$xy = 4$ so $y = \frac{4}{x}$

Sum of squares $= S = x^2 + y^2 = x^2 + \left(\frac{4}{x}\right)^2 = x^2 + \frac{16}{x^2}$

$S' = 2x - \frac{32}{x^3} = 0$

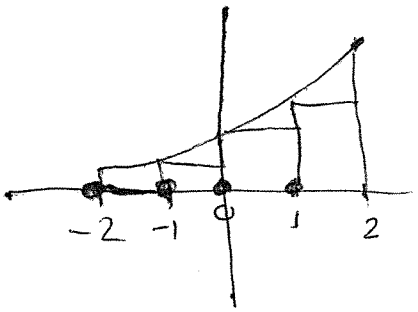
$2x = \frac{32}{x^3} \rightarrow 2x^4 = 32 \rightarrow x^4 = 16 \rightarrow x = 2$ (Since $x > 0$)

Since $S' = \frac{2x^4 - 32}{x^3}$ is < 0 if $0 < x < 2$, > 0 if $2 < x$, $x = 2$ gives an abs. min value for S.

$x = 2 \rightarrow y = 2$

So the smallest possible S is $2^2 + 2^2 = 8$

2. For $y = f(x) = e^x$, what is the "right end-point approximation" L_4 to the area under the graph and over the interval $[-2, 2]$? Does L_4 overestimate or underestimate the exact area A? why?



$\Delta x = \frac{2 - (-2)}{4} = 1$

$L_4 = \Delta x \left(f(-2) + f(-1) + f(0) + f(1) \right)$

$= 1 \left(e^{-2} + e^{-1} + e^0 + e^1 \right)$

$= 1 + e + \frac{1}{e} + \frac{1}{e^2}$

Since f is increasing, L_4 underestimates A.