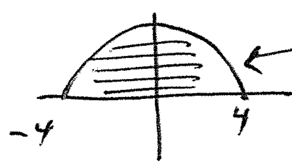
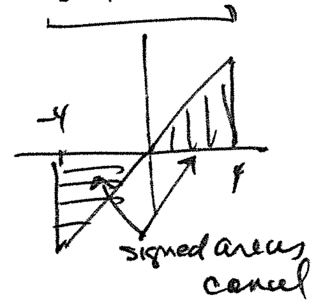
Math 131, Fall 2016
Quiz 9, December 8, 2016
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.
Since you have no calculator for the quiz, your final answers might contain unevaluated expressions like $\frac{\ln 7}{6 \ln 2}$ *or* e^{-1}.

1. Using properties of the integral and areas (not the Fundamental Theorem of Calculus), find the value of $\int_{-4}^{4} \sqrt{16 - x^2} + x \, dx$.

$$\int_{-4}^{4} \sqrt{16-x^2} + x \, dx = \int_{-4}^{4} \sqrt{16-x^2} \, dx + \int_{-4}^{4} x \, dx$$





$$= \frac{1}{2}(\pi \cdot 4^2) = 8\pi$$

2. On what open interval is the function $F(x) = \int_{3}^{x} (t+1)\sqrt{2 + \sin t} \, dt$ decreasing?

$F'(x) = (x+1)\sqrt{2+\sin x} = 0$ when $x = -1$

($\sqrt{2+\sin x}$ always > 0)

sign of $F'(x)$: $F'(x) < 0$ | -1 | $F'(x) > 0$

$\therefore$ $F(x)$ is decreasing on the open interval $(-\infty, -1)$

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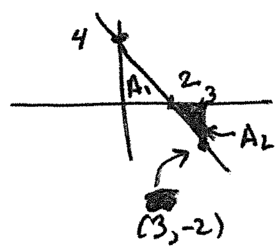
1. Using properties of the integral and areas (not the Fundamental Theorem of Calculus), find the value of $\int_3^0 4 - 2x\, dx$.

$$\int_3^0 4-2x\, dx = -\int_0^3 4-2x\, dx = -\left[\text{Area } A_1 - \text{Area } A_2\right]$$

$$= -\left[\frac{1}{2}(2)(4) - \frac{1}{2}(1)(2)\right]$$

$$= -\left[4-1\right] = -3$$



2. On what open interval is the function $F(x) = \int_{-1}^{x} t^2 + 2t + 7\, dt$ concave down?

$$F'(x) = \left(\int_{-1}^{x} t^2+2t+7\, dt\right)' = x^2+2x+7$$

$$F''(x) = 2x+2 = 0 \text{ when } x=-1$$

Sign of $F''(x)$: $F''(x)<0$ | -1 | $F''(x)>0$

$F(x)$ is concave down on the open interval $(-\infty,-1)$

Math 131, Fall 2016
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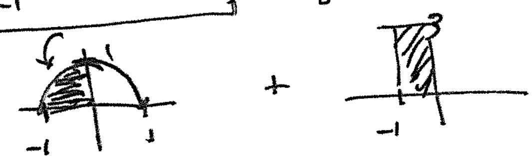
1. Using properties of the integral and areas (not the Fundamental Theorem of Calculus), find the value of $\int_{-1}^{0} 4\sqrt{1-x^2} + 3\,dx$

$$\int_{-1}^{0} 4\sqrt{1-x^2}+3\ dx = 4\int_{-1}^{0}\sqrt{1-x^2}\ dx + \int_{-1}^{0} 3\,dx$$



$$= 4\left(\frac{1}{4}\pi(1)^2\right) + 3(1) = \pi + 3$$

2. Suppose that $\int_{-2}^{2} f = 4$, $\int_{1}^{3} f = 1$ and $\int_{-2}^{3} f = 8$. What is $\int_{1}^{2} f$?

$$\int_{-2}^{2} f + \int_{2}^{1} f + \int_{1}^{3} f = \int_{-2}^{3} f$$

$$4 + \int_{2}^{1} f + 1 = 8$$

$$\int_{2}^{1} f = 3$$

$$\int_{1}^{2} f = -\int_{2}^{1} f = -3$$

Math 131, Fall 2016
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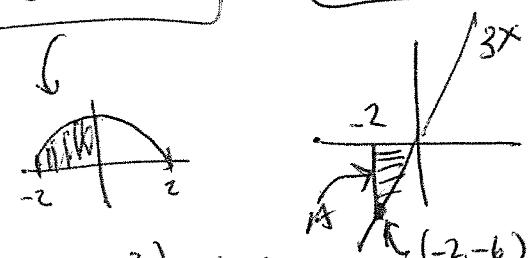
1. Using properties of the integral and areas (not the Fundamental Theorem of Calculus), find the value of $\int_{-2}^{0} \sqrt{4-x^2} - 3x\,dx$.

$$\int_{-2}^{0} \sqrt{4-x^2} - 3x\,dx = \int_{-2}^{0} \sqrt{4-x^2}\,dx - \int_{-2}^{0} 3x\,dx$$

$\int_{-2}^{0} 3x\,dx = -A$



$$= \frac{1}{4}(\pi \cdot 2^2) - (-6)$$

$$= \pi + 6$$

2. Where does the function $F(x) = \int_1^x 3t^3 - 81t\,dt$ have inflection points?

$F'(x) = 3x^3 - 81x$

$F''(x) = 9x^2 - 81 = 0$

$x^2 = 9$

$x = \pm 3$

Sign of $F''(x) = 9(x-3)(x+3)$:

$F''(x) > 0$ for $x < -3$; $F''(x) < 0$ for $-3 < x < 3$; $F''(x) > 0$ for $x > 3$

So F changes concavity at $x = -3$ and at $x = 3$

inflection points for F

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1. Using properties of the integral and areas (not the Fundamental Theorem of Calculus), find the value of $\int_{-2}^{0} 7|x+1|\,dx$

$$\int_{-2}^{0} 7|x+1|\,dx = 7\int_{-2}^{0} |x+1|\,dx$$

$$|x+1| = \begin{cases} x+1 & \text{if } x \geq -1 \\ -x-1 & \text{if } x \leq -1 \end{cases}$$

$$= 7\left(\tfrac{1}{2} + \tfrac{1}{2}\right) = 7$$

2. Suppose f is continuous and that $-3 \leq f(x) \leq 2$ on the interval $[5,7]$.

What are the largest and smallest possible values for $\int_5^7 f(x)\,dx$?

$-3 \leq f(x) \leq 2$ so

$$\int_5^7 -3\,dx \leq \int_5^7 f(x)\,dx \leq \int_5^7 2\,dx$$

$$-6 = -3(7-5) \leq \int_5^7 f(x)\,dx \leq 2(7-5) = 4$$