Background expectations for students in Calculus I (Math 131)

The following list illustrates the kind of background skills and techniques from algebra, trig, and precalculus that you should have for Calculus I. You probably shouldn't worry too much if one or two items seem unfamiliar – you can fill in as needed – but you really do need most of these skills at one time or another in the course.

In addition to these skills, you need the concentration to read carefully and interpret what you read for the purpose of solving problems.

<u>In problems that come up in calculus, it's often true that more of the actual work depends on precalculus skills than on calculus tools</u>. You should be able to do these things without using a calculator.

For Fall 2016, the Residential Peer Mentors (RPMs) for Calc I have been alerted to expected questions about some of the example questions below (as well as on the precalculus Diagnostic Tests in the textbook on pp. xxvi - xxx). The RPMs schedules are available at the Cornerstone web page.

1. Be able to manipulate algebraic expressions, including correct calculations with fractions and using the rules of exponents.

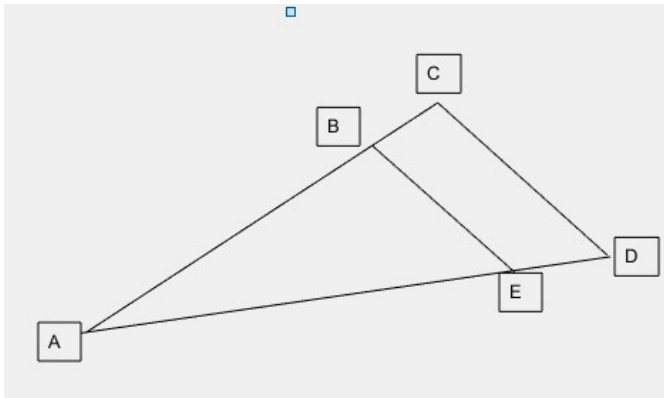
Examples: Write $\frac{a+c}{c} - \frac{c}{a+c}$ as a single fraction and simplify as much as possible.

Simplify $(\sqrt[3]{x})^{-\frac{1}{2}}\sqrt[6]{x}$ as much as possible.

2. Know the basic properties of plane and solid geometric figures – for example, areas of a triangle, parallelogram and circles; area of a sector in a circle; volume of a rectangular box, sphere, cylinder. Know the Pythagorean Theorem and be apply to apply it.
Be able to recognize and use proportional relationships derived from similar triangles.

Examples: the points $P = (0,0)$, $Q = (1,1)$, $R = (2,0)$ and $S = (3,1)$ are the vertices of a parallelogram. What is its area? What is the area of $\triangle PQS$? What is the area and the circumference of the circle that has the line segment RS as a radius?

In the figure below, BE and CD are parallel. If the product of the lengths of AB and CD is 5, and the length of BE is 4, what is the length of AC?



4. Be able to determine the intersection of two straight lines, or a line and a quadratic function. Be able to find an equation for a line. Be able to determine whether two lines are parallel or perpendicular suing their slopes.

Examples: Where do the lines $3x + 2y = 4$ and $-x + 4y = 2$ intersect? Are these lines perpendicular?

Where do the lines $y - 2x + 5 = 0$ and $2y - 3 = 4x$ intersect?

What is the equation of the straight line through $(1, 1)$ and perpendicular to the line $y = \frac{1}{2}x$?

5. Be able to solve linear equations and quadratic equations (by factoring, or by using the quadratic formula, when needed). Be able to solve linear and quadratic inequalities.

Examples: The inequality $1 \leq 5 - 2x \leq 4$ is true for what values of x? For what x's is the inequality $x^2 + x - 2 < 0$ true?

Where do the line $y = 2x$ and the parabola $y = x^2 + x - 2$ intersect?

6. Be able to complete the square of a quadratic expression and recognize when completion the square is appropriate.

7. Be able to graph quadratic functions, ellipses, circles, hyperbolas.

Examples: Draw the graphs of

i) $x^2 - 4x + y^2 + 2y = -2$
ii) $9x^2 - 18x + 4y^2 + 8y = 23$
iii) $yx - y = 4$

8. Understand the idea of a function and function notation including the composition of functions. Be able to recognize the functions from which a given function is composed.

Examples: i) For $f(x) = x^2$, simplify $\frac{f(x+h)-f(x)}{h}$ as much as possible.

ii) For what functions f, g, h is $f(g(h(x) = \sin^3(x^2)$
For the same three functions f, g, h:

$$h(g(f(x))) =$$
$$g(f(h(x))) =$$
$$f(h(g(x))) =$$

iii) If you have a picture of the graph of $y = f(x)$, how would you get a picture of the graph of $y = f(x+5)$? How would you get a picture of the graph of $y = -2f(x)$?

9. Be able to determine the domain of a function.

10. Be familiar with the graphs of the logarithmic and exponential functions

Examples: i) What is the domain of $g(x) = \frac{x-1}{(x-1)(x+2)(x^2+x+1)}$. Is it different from the domain of $f(x) = \frac{1}{(x+2)(x^2+x+1)}$?

ii) What is the domain of $f(x) = \log_2(2x+5)$?

ii) Draw a reasonable picture of the graph of $y = g(x) = 2^{(x-2)}$

11. Be able to use the properties of logarithmic function to simplify expressions or solve equations.

Examples: i) Simplify $\log\left(\left(\frac{100}{1000} * 2000\right)\right)^{1/2}$ (Here, "log" means $\log_{10}$)
ii) Using logarithms, solve the equation $2^{5x+1} = 7$

12. Know the definitions of the trigonometric functions, be familiar with their graphs and periodicity, be able to evaluate trigonometric functions using standard triangles, and know the basic trigonometric identities including the Law of Cosines.

Examples: i) What are $\sin 45°$, $\cos 45°$ and $\tan 45°$?
ii) What are $\sin\theta$ and $\cos\theta$ if $\theta = \frac{\pi}{6}$ radians
iii) If $\sin\theta = \frac{1}{3}$, what is $\sin(2\theta)$?

iv) Find $\sin(75°)$, using that $75° = 30° + 45°$

iv) If the three sides of a triangle have lengths $2, 3,$ and $4,$ then what is the angle between the sides that have lengths 2 and 3? (approximate answer, correct to one decimal place)