Limits Involving "Infinity"

For numbers a and L, it makes sense to say

"x gets close to a" or that "$f(x)$ gets close to L"

"∞" isn't a number, and it doesn't make sense to say

"x gets close to ∞" or that "$f(x)$ gets close to ∞"

instead we say

"x gets larger than any given number N" or that "$f(x)$ gets larger than any given number N"

So then

$\lim_{x \to a} f(x) = \infty$ means: we can make $f(x)$ larger than any given number N by making x close enough to a

$\lim_{x \to a} f(x) = -\infty$ means: we can make $f(x)$ smaller than any given number $(-N)$ by making x close enough to a.

(*with similar meanings for* $x \to a^+$ *or* $x \to a^-$)

$\lim_{x\to\infty} f(x) = L$ means: we can make $f(x)$ as close to L as we like by making x large enough

that is: for any given $\epsilon > 0$, it will eventually be true that $|f(x) - L| < \epsilon$; it will be true after x gets bigger than some number N

$\lim_{x\to-\infty} f(x) = L$ means: we can make $f(x)$ as close to L as we like by making x small enough (negative)

that is: for any given $\epsilon > 0$, it will eventually be true that $|f(x) - L| < \epsilon$; it will be true after x gets smaller than some number $(-N)$.

$\lim_{x \to \infty} f(x) = \infty$ means: we can make $f(x)$ larger than any given number N by making x large enough

that is: for any given number N, it will eventually be true that $f(x) > N$; it will be true after x gets bigger than some number M

$\lim_{x \to -\infty} f(x) = \infty$ means: we can make $f(x)$ larger than any given number N by making x small enough (negative)

that is: for any given number N, it will eventually be true that $f(x) > N$; it will be true after x gets smaller than some number $(-N)$

$\lim_{x \to \infty} f(x) = -\infty$ means: we can make $f(x)$ smaller than any given number $(-N)$ by making x large enough

that is: for any given number $(-N)$, it will eventually be true that $f(x) < -N$; it will be true after x gets bigger than some number M

$\lim_{x \to -\infty} f(x) = -\infty$ means: we can make $f(x)$ smaller than any given number $(-N)$ by making x small enough (negative)

that is: for any given number $(-N)$, it will eventually be true that $f(x) < -N$; it will be true after x gets smaller than some number $(-M)$