Example About Interpreting the Product Rule
(*taken from Exercise 49, p. 190*)

$P(t)$ represents the population of the Richmond-Petersburg, VA metro area at time t years: $t = 0$ is 1999.

$A(t)$ represents the average income (per capita) for persons in that region at time t

units:	t	years
	$P(t)$	persons
	$A(t)$	$/person

$T(t) = P(t)A(t) =$ total personal income for that region

units: $T(t)$ persons · $/person = $29412110200

Census data gives:

$P(0) = 961400$ persons $A(0) = 30593$ $/person $T(0) =$ $29, 412,110,200

$\frac{dP}{dt}\big|_{t=0} = 9200 \ \frac{\text{persons}}{\text{year}}$ $\qquad$ $\frac{dA}{dt}\big|_{t=0} = 1400 \ \frac{\text{\$/person}}{\text{year}}$

$T(t)$ changes with time. It changes partly because $P(t)$ changes and partly because $A(t)$ changes. What is the rate of change of $T(t)$ with respect to t? The Product Rule states that

$$\frac{dT}{dt} = P(t)\frac{dA}{dt} + A(t)\frac{dP}{dt}$$

and evaluating each quantity when $t = 0$ gives:

$$\frac{dT}{dt}\Big|_{t=0} = (961400)\ (1400) \quad + \quad (30593)\ (9200) = 1{,}627{,}415{,}600$$

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Units: $/year $\qquad$ (persons)$\left(\frac{\text{\$/person}}{\text{year}}\right)$ + ($/person)(persons/year)

So, at time $t = 0$ (1999), the total personal income in the area is about $29.4 billion and it is changing at a rate of about $1.6 billion/year.