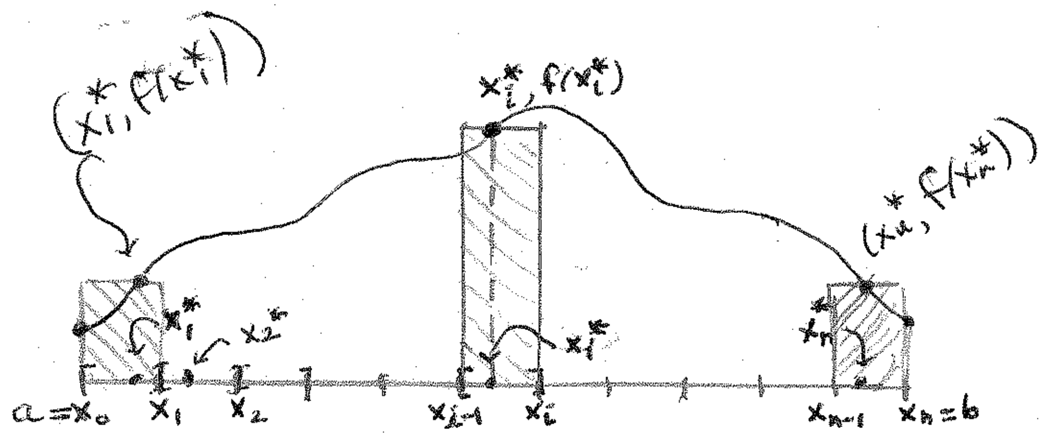

Pick an n. Divide $[a, b]$ into n equal subintervals, each one with length $\Delta x = \frac{b-a}{n}$.
The division points are $a = x_0,\ x_1, \ldots,\ x_i,\ \ldots,\ x_{n-1},\ x_n$

Pick a "sample point" in each subinterval: x_1^* in the first interval $[a, x_1] = [x_0, x_1]$,
x_2^* in the second interval $[x_1, x_2]$
$\vdots$
x_i in the i^{th} interval $[x_{i-1}, x_i]$
$\vdots$
x_n in the n^{th}subinterval $[x_{n-1}, x_n] = [x_{n-1}, b]$

On each subinterval $[x_{i-1}, x_i]$ build a rectangle with height $= f(\textit{sample point}) = f(x_i^*)$
Add up the areas of these rectangles:

$$Z_n = f(x_1^*)\Delta x + f(x_2^*)\Delta x + \ldots + f(x_i^*)\Delta x + \ldots + f(x_n^*)\Delta x = \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$(*)$ <u>If we choose the right endpoints as the sample points</u> (that is, $x_i^* = x_i$), we get the <u>right enpoint Riemann sum</u>

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_i)\Delta x + \ldots + f(x_n)\Delta x = \sum_{i=1}^{n} f(x_i)\Delta x$$

$(**)$ <u>If we choose the left endpoints as the sample points</u> (that is, $x_i^* = x_{i-1}$), we get the <u>left enpoint Riemann sum</u>

$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + \ldots + f(x_{i-1})\Delta x + \ldots + f(x_{n-1})\Delta x = \sum_{i=1}^{n} f(x_{i-1})\Delta x$$

$(***)$ <u>If we choose the midpoints as the sample points</u> (that is, $x_i^* = m_i$), we get the <u>midpoint Riemann sum</u>

$$M_n = f(m_1)\Delta x + f(m_2)\Delta x + \ldots + f(m_i)\Delta x + \ldots + f(m_n)\Delta x = \sum_{i=1}^{n} f(m_i)\Delta x$$

If $y = f(x) \geq 0$ on $[a, b]$ and f is continuous, then $\lim\limits_{n\to\infty} R_n = \lim\limits_{n\to\infty} L_n$ $= \lim\limits_{n\to\infty} M_n = \lim\limits_{n\to\infty} Z_n =$ area under graph of $y = f(x)$ above $[a, b]$