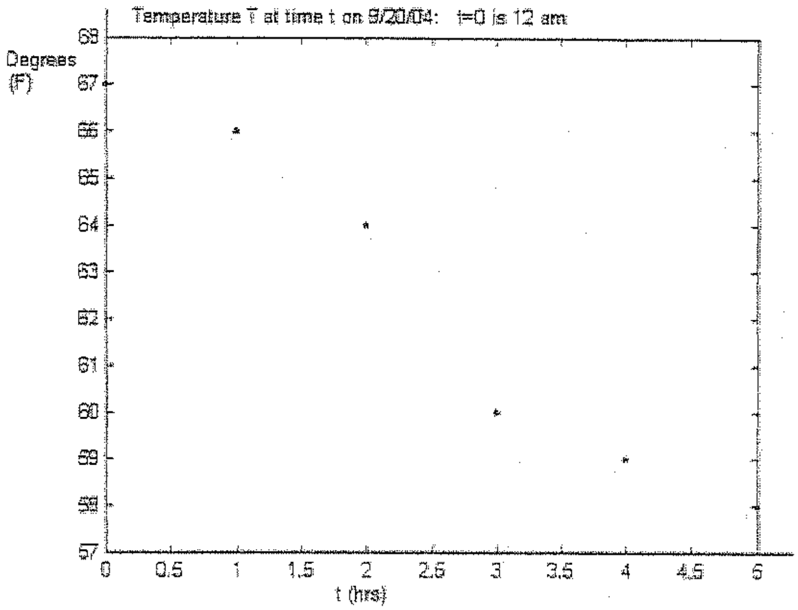
The Derivative of a Temperature Function

http://www.math.wustl.edu/~freiwald/~~Math131/temperature.pdf~~ /131temperature.pdf

A temperature gauge at Lambert Airport on September 20, 2004 is periodically monitoring the temperature T (in F°) at times t (hrs) starting at 12:00 am ($t = 0$). Of course the temperature is a function of time: $T(t)$. Here is some of the data collected.



t	T
0	67
1	66
2	64
3	60
4	59
5	58

$T'(t)$ is the rate of change of the temperature with respect to time. Its <u>units</u> are F°/hr
$T'(3)$ is the rate of change at 3 am. $T'(3) = ?$

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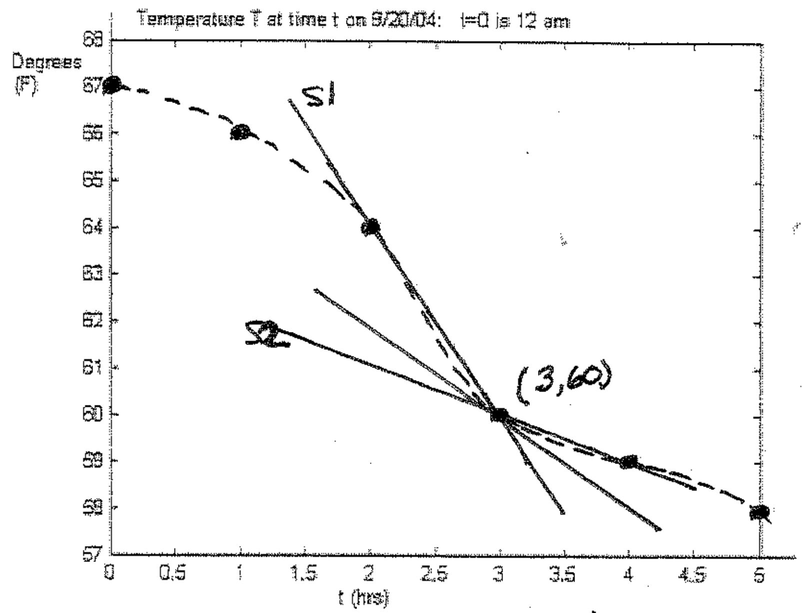

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1	66
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$$T'(3) = \lim_{t \to 3} \frac{T(t) - T(3)}{t - 3} = \ ? \qquad (F°/hr)$$

$T'(3) \approx$ slope of secant line S1
$T'(3) \approx$ slope of secant line S2

two estimates:
- one looks too big
- one looks too small

so

$T'(t)$ is the rate of change of the temperature with respect to time. Its <u>units</u> are F°/hr
$T'(3)$ is the rate of change at 3 am. $T'(3) = \ ?$

We could <u>estimate</u> the value of $T'(3)$ by averaging estimates made with the most "nearby" data:

$$T'(3) \approx \tfrac{1}{2}\left(\tfrac{60-64}{3-2} + \tfrac{59-60}{4-3}\right) = \tfrac{1}{2}(-4-1) = -\tfrac{5}{2} \text{ F°/hr}$$

We could <u>estimate</u> the value of $T'(4)$ in the same way:

$$T'(4) \approx \tfrac{1}{2}\left(\tfrac{59-60}{4-3} + \tfrac{58-59}{5-4}\right) = \tfrac{1}{2}(-1-1) = -1 \text{ F°/hr}$$

If we needed an estimate for $T'(3.5)$ (that is, at 3:30 am), our best guess would probably be

$$T'(3.5) \approx \tfrac{1}{2}(T'(3) + T'(4)) = \tfrac{1}{2}\left(-\tfrac{5}{2} - 1\right) = -\tfrac{7}{4} \text{ F°/hr}$$