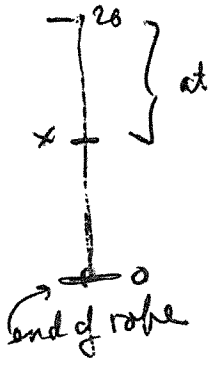
Math 132, Spring 2017
Quiz 2, February 14, 2017
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.

1. A 20 ft rope that weighs 2 lbs/ft hangs over the side of a building. (*Nothing is attached to the rope.*) How much work is done in lifting the rope to the top of the building. (*Include the units on your answer.*)



at ht x, rope remaining to lift = $20-x$ ft
weighing $(20-x)2$ lbs

$$W = \int_0^2 2(20-x)\,dx = \int_0^2 40-2x\,dx$$

$$= 40x - x^2 \Big|_0^{20} = 400 \text{ ft-lbs}$$

2. The average value of $f(x) = ax + 3$ over the interval $[0, 4]$ is 9. What is a?

$$9 = \operatorname{avg}_{[0,4]} f = \frac{1}{4-0}\int_0^4 ax+3\,dx = \frac{1}{4}\left[a\frac{x^2}{2}+3x\right]_0^4$$

$$= \frac{1}{4}\left[\quad 8a+12\right] = 2a+3$$

so $2a = 6$

$a = 3$

Math 132, Spring 2017
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For all 9 a.m. Sections
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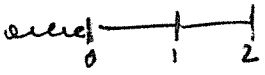
1. The force needed to hold a spring stretched to 1 m beyond its natural length is $\frac{1}{2}$N. How much work is necessary to stretch it an additional meter? (*Include units on the answer.*)

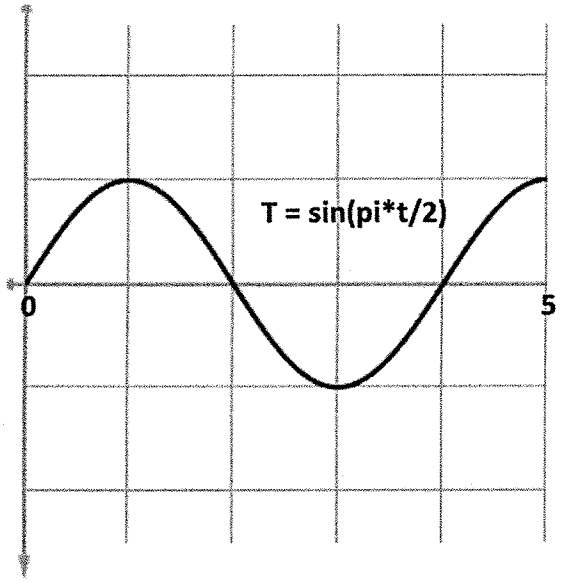
Hooke's Law: $F(x) = kx$



$\frac{1}{2} = F(1) = k \cdot 1 \rightarrow k = \frac{1}{2}$

$$W = \int_1^2 \underbrace{\frac{1}{2}x}_{F(x)}\,dx = \frac{x^2}{4}\Big]_1^2 = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}\text{J}.$$

2. On the planet Krypton, the measured temperature T at time t $(0 \le t \le 5)$ is $T(t) = \sin\left(\frac{\pi t}{2}\right)$ C°.



$$\text{avg } T = \frac{1}{5-0}\int_0^5 \sin\left(\frac{\pi t}{2}\right)dt$$

$[0,5]$

$$= \frac{1}{5}\left[-\frac{2}{\pi}\cos\left(\frac{\pi t}{2}\right)\right]_0^5$$

$$= \frac{1}{5}\cdot\frac{-2}{\pi}\left[\cos\left(\frac{5\pi}{2}\right) - \cos 0\right]$$

$$= \frac{-2}{5\pi}\left[0 - 1\right] = \frac{2}{5\pi}\ (\text{C}^\circ)$$

What is the average temperature over this time period?

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For all 10 a.m. Sections
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1. It takes 20 ft-lbs of work to stretch a certain spring 5 ft beyond its natural length. What was the force $F(x)$ required at each value x $(0 \le x \le 5)$ to continue stretching ? (*Include units on the answer.*)

Hooke's Law: $F(x) = kx$ = force required at x

$$20 = \int_0^5 kx\,dx = k\,\frac{x^2}{2}\Big|_0^5 = k \cdot \frac{25}{2}$$

so $k = \frac{20 \cdot 2}{25} = \frac{8}{5}$

$F(x) = \frac{8}{5}x$ (lbs)

2. The average value of the function $f(x) = k\cos x$ over the interval $[0, \frac{3\pi}{2}]$ is 3. What is k?

$$3 = \text{avg } f = \frac{1}{\frac{3\pi}{2} - 0} \int_0^{3\pi/2} k\cos x\,dx$$

$$3 = \frac{2}{3\pi} \int_0^{3\pi/2} k\cos x = \frac{2}{3\pi}\left[k\sin x \right]_0^{3\pi/2}$$

$$= \frac{2}{3\pi}\left[-k\right]$$

so $k = -\frac{3(3\pi)}{2}$

$k = -\frac{9\pi}{2}$

Math 132, Spring 2017
Quiz 2, February 14, 2017
For all 11 a.m. Sections
Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.

1. It takes 1 N of force to hold a spring compressed to 1 m shorter than its natural length. How much work would be required to compress it an additional meter? (*Include units on the answer.*)

$F(x) = kx$

$F(1) = k \cdot 1 \rightarrow k = 1$

$$W = \int_1^2 F(x)\,dx = \int_1^2 x\,dx = \frac{x^2}{2}\Big|_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}\ \text{J}$$

2. Suppose $a > 0$. The average value of the function x^2 on the interval $[0, a]$ is 4. What is a?

$$4 = \underset{[0,a]}{\text{avg}}\ x^2 = \frac{1}{a-0}\int_0^a x^2 = \frac{1}{a}\left(\frac{x^3}{3}\right)\Big|_0^a$$

$$= \frac{1}{a} \cdot \frac{a^3}{3}$$

$$= \frac{a^2}{3}$$

$\therefore a^2 = 12$

$a = \sqrt{12} = 2\sqrt{3}$

↑ since $a > 0$

Math 132, Spring 2017
Quiz 2, February 14, 2017
For all 12 p.m. Sections
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1. Use an integral to compute the average value of the function $f(x) = \frac{1}{x^2}$ over the interval $[1, 3]$.

$$\text{avg}_{[1,3]} f = \frac{1}{3-1}\int_1^3 \frac{1}{x^2}\,dx = \frac{1}{2}\left(-\frac{1}{x}\right)\Big]_1^3 = \frac{1}{2}\left[-\frac{1}{3}+1\right] = \frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$$

2. If 2 J of work are required to stretch a spring 4 meters beyond its natural length, how much work would be required to stretch it only 1 m beyond its natural length? (*Include units on the answer.*)

Hooke's Law: $F(x) = kx$

$$2 = W = \int_0^4 kx\,dx = k\frac{x^2}{2}\Big]_0^4 = k\left[\frac{16}{2} - 0\right] = 8k$$

So $k = 1/4$

$$\therefore W = \int_0^1 \frac{1}{4}x\,dx = \frac{1}{8}x^2\Big]_0^1 = \frac{1}{8}\text{ J.}$$