Math 132, Spring 2017
Quiz 5 March 21, 2017
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
<u>Do NOT use any methods except those discussed so far in this course.</u>

1. Write the formula for the Simpson approximation S_8 for the integral $\int_0^{16} f(x)\,dx$.
(*In the formula, fill in as many specific values as you can; since you don't have a formula for* f, *you can just write* $f(0)$ *for the value of* f *at* 0, *etc.*)

$$\Delta x = \frac{16-0}{8} = 2$$

0 2 4 6 8 10 12 14 16

$$S_8 = \frac{2}{3}\big(f(0) + 4f(2) + 2f(4) + 4f(6) + 2f(8) + 4f(10) + 2f(12) + 4f(14) + f(16) \big)$$

2. The following integral is defined to be a certain limit. Write the limit and the decide whether the integral converges or diverges. If it converges, find its value.

$$\int_{-\infty}^{-1} \frac{x}{1+x^2}\,dx$$

$$\int_{-\infty}^{-1} \frac{x}{1+x^2}\,dx = \lim_{t\to-\infty} \int_t^{-1} \frac{x}{1+x^2}\,dx = \lim_{t\to-\infty} \frac{1}{2}\ln(1+x^2)\Big|_t^{-1}$$

$$= \lim_{t\to-\infty} \frac{1}{2}\ln 2 - \underbrace{\frac{1}{2}\ln(1+t^2)}_{\downarrow\ -\infty} \quad \text{d.n.e. (or, } = -\infty)$$

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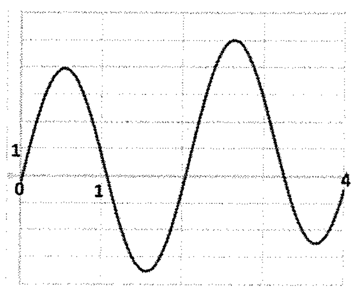
1. The following is graph of the **fourth derivative** of $y = f(x)$:



S_{10} is the Simpson approximation for $\int_0^4 f(x)\,dx$. We can estimate that

$$\left|\int_0^4 f(x)\,dx - S_{10}\right| \le ???$$

(*You may leave your answer in fraction form; no need to convert to a decimal.*)

From graph, $|f^{(4)}(x)| \le 5$ for $0 \le x \le 4$, so

$$\left|\int_0^4 f(x)\,dx - S_{10}\right| \le \frac{K(b-a)^5}{180n^4} = \frac{5(4-0)^4}{180(10)^4} \quad \left(= \frac{2^2}{9 \cdot 5^4}\right)$$

2. The following integral is defined to be a certain limit. Write the limit and the decide whether the integral converges or diverges. If it converges, find its value.

$$\int_{-\infty}^{\infty} \frac{3}{2+2x^2}\,dx = \lim_{t\to-\infty} \int_t^0 \frac{3}{2+2x^2}\,dx + \lim_{t\to\infty} \int_0^t \frac{3}{2+2x^2}\,dx$$

< note: "breaking" the integral into 2 parts at 0 was an arbitrary choice >

$$= \lim_{t\to-\infty} \frac{3}{2}\int_t^0 \frac{1}{1+x^2}\,dx + \lim_{t\to\infty} \frac{3}{2}\int_0^t \frac{1}{1+x^2}\,dx$$

$$= \lim_{t\to-\infty} \frac{3}{2}(\arctan x)_t^0 + \lim_{t\to\infty} \frac{3}{2}(\arctan x)_0^t$$

$$= \frac{3}{2}\cdot\frac{\pi}{2} + \frac{3}{2}\cdot\frac{\pi}{2} = \frac{3\pi}{2}$$

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1. S_6 is Simpson's approximation to $\int_0^2 \sin x \, dx$ using $n = 6$. WE can estimate that

$$\int_0^2 \sin x \, dx - S_6 \,| \leq ???$$

(*You may leave you answer in fraction form; no need to convert to a decimal.*)

Since $|\sin^{(4)}(x)| = |\sin x| \leq 1$ for all x,

$$\left| \int_0^2 \sin x \, dx - S_6 \right| \leq \frac{K(b-a)^5}{180 n^4} = \frac{1 \cdot (2)^5}{180 \cdot 6^4} \quad \left(= \frac{1}{10 \cdot 3^6} \right)$$

2. The following integral is defined to be a certain limit. Write the limit and the decide whether the integral converges or diverges. If it converges, find its value.

$$\int_{-\infty}^{-1} \frac{1}{x^3} \, dx$$

$$\int_{-\infty}^{-1} \frac{1}{x^3} dx = \lim_{t \to -\infty} \int_t^{-1} \frac{1}{x^3} dx = \lim_{t \to -\infty} \left(\frac{-1}{2x^2} \right) \Big|_t^{-1}$$

$$= \lim_{t \to -\infty} \left(-\frac{1}{2} + \frac{1}{2t^2} \right) = -\frac{1}{2} + 0 = -\frac{1}{2}$$

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For all 11 a.m. Sections
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1. A point moves along a straight line. Suppose you have the following velocity information:

t (sec)	$v(t)$ (m/sec)
2	1
3	3
4	1
5	3
6	1
7	3
8	1

Use this data and Simpson's Rule to estimate the distance traveled for $2 \leq t \leq 8$.

$[2,8]$ is ~~divided~~ divided into $n=6$ parts, each of length $\Delta x = \frac{8-2}{6} = 1$. Since $v(t) \geq 0$,

Distance travelled $= \int_0^8 v(t)\,dt \simeq S_6 = \frac{1}{3}\left(f(2)+4f(3)+2f(4)+4f(5)+2f(6)+4f(7)+f(8)\right)$

$= \frac{1}{3}(1+12+2+12+2+12+1)$

$= \frac{1}{3}(42) = 14$ (m)

2. The following integral is defined to be a certain limit. Write the limit and the decide whether the integral converges or diverges. If it converges, find its value.

$$\int_0^{\infty} xe^{-x}\,dx$$

Note $\int \underset{u}{x}\,\underbrace{e^{-x}dx}_{dv} = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C$

$du = dx \quad v = -e^{-x}$

So $\int_0^{\infty} xe^{-x}dx = \lim_{t\to\infty} \int_0^t xe^{-x}dx = \lim_{t\to 0} \left(-xe^{-x} - e^{-x}\right)\Big|_0^t$

$= \lim_{t\to 0} \left(-te^{-t} - e^{-t}\right) + 1$

$= \lim_{t\to 0} \left(\frac{-t}{e^t} - \frac{1}{e^t}\right) + 1 = 1$

$\quad \downarrow 0 \quad \downarrow 0$

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1. Suppose you know that the 4th derivative $f^{(\text{iv})}(x) = \frac{3}{x^2}$ for $x > 0$. Let S_{10} be the Simpson approximation for $\int_1^3 f(x)\,dx$ with $n = 10$. We can estimate that

$$|\int_1^3 f(x)\,dx - S_{10}| \leq ???$$

(*You may leave your answer in fraction form; no need to convert to a decimal.*)

on $[1,3]$, $|f^{4}(x)| = \left|\frac{3}{x^2}\right| \leq \frac{3}{1^2} = 3$

So $\left|\int_1^3 f(x)\,dx - S_{10}\right| \leq \frac{K(b-a)^5}{180n^4} = \frac{3(2^5)}{180 \cdot 10^4} \left(= \frac{1}{30 \cdot 5^4}\right)$

2. The following integral is defined as a certain limit. Write the limit and the decide whether the integral converges or diverges. If it converges, find its value.

$$\int_1^\infty \frac{3x}{1+x^2}\,dx$$

$\int_1^\infty \frac{3x}{1+x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{3x}{1+x^2}\,dx$

$= \lim_{t\to\infty} \frac{3}{2}\ln(1+x^2)\Big|_1^t = \lim_{t\to\infty} \Big(\underbrace{\tfrac{3}{2}\ln(1+t^2)}_{\downarrow\ \infty} - \tfrac{3}{2}\ln 2\Big)$ d.n.e. $(\text{or} = \infty)$