Math 132, Spring 2017
Quiz 6 March 28, 2017
For all 8 a.m. Sections
Show enough work to make it clear how you got your answer.
<u>Do NOT use any methods except those discussed so far in this course.</u>

1. Does $\int_0^1 \frac{1}{\sqrt[3]{x^2}}\,dx$ converge or diverge? If it converges, find its value. (*Be sure to include a step that shows the <u>limit</u> used to answer the question.*)

$$\int_0^1 \frac{1}{\sqrt[3]{x^2}}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{1}{\sqrt[3]{x^2}}\,dx = \lim_{t\to 0^+} 3x^{\frac{1}{3}}\Big|_t^1$$

$$= 3 - 0 = 3$$

2. Let $a_n = \frac{3+5n^2}{1+n}$. Does $\lim_{n\to\infty} a_n$ exist? If it does, find its value.

Limit does not exists: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3+5n^2}{1+n} = \lim_{n\to\infty} \frac{\frac{3}{n^2}+5}{\frac{1}{n^2}+\frac{1}{n}}$ d.n.e. (or, $=\infty$)

$\to 5$

$\to 0$

Math 132, Spring 2017
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For all 9 a.m. Sections
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1. Does $\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$ converge or diverge? If it converges, find its value. (*Be sure to include a step that shows the <u>limit</u> used to answer the question.*)

$$\int_0^1 \frac{x}{\sqrt{1-x^2}} = \lim_{t\to 1^-} \int_0^t \frac{x}{\sqrt{1-x^2}} = \lim_{t\to 1^-} -\sqrt{1-x^2}\Big|_0^t$$

$(u = 1-x^2)$

$$= 0 - (-1) = 1$$

2. Let $a_n = \frac{3\sqrt{n}}{\sqrt{n}+2}$. Does $\lim\limits_{n\to\infty} a_n$ exist? If it does, find its value.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3\sqrt{n}}{\sqrt{n}+2} = \lim_{n\to\infty} \frac{3}{1+2/\sqrt{n}} = 3$$

Math 132, Spring 2017
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For all 10 a.m. Sections
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1. Does $\int_0^6 \frac{1}{x-5}\,dx$ converge or diverge? If it converges, find its value. (*Be sure to include a step that shows the <u>limit</u> used to answer the question.*)

$$\int_0^6 \frac{1}{x-5}\,dx = \int_0^5 \frac{1}{x-5}\,dx + \int_5^6 \frac{1}{x-5}\,dx$$ (provided both integrals on the right converge)

$$\int_0^5 \frac{1}{x-5}\,dx = \lim_{t\to 5^-} \int_0^t \frac{1}{x-5}\,dx$$
$$= \lim_{t\to 5^-} \ln|x-5|\Big|_0^t = \lim_{t\to 5^-} \underbrace{\ln|t-5|}_{\to -\infty} - \ln 5 : \text{dne.}$$ (or, $= -\infty$)

Since $\int_0^5 \frac{1}{x-5}dx$ diverges, so does $\int_0^6 \frac{1}{x-5}dx$

2. Let $a_n = \sqrt{\frac{1+4n^2}{1+n^2}}$. Does $\lim\limits_{n\to\infty} a_n$ exist? If it does, find its value.

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1. Does $\int_0^{16} \frac{1}{\sqrt[4]{x}}\,dx$ converge or diverge? If it converges, find its value. (*Be sure to include a step that shows the* *limit* *used to answer the question.*)

$$\int_0^{16} \frac{1}{\sqrt[4]{x}}\,dx = \lim_{t\to 0^+} \int_t^{16} x^{-1/4}\,dx = \lim_{t\to 0^+} \frac{4}{3}x^{3/4}\Big|_t^{16}$$

$$= \frac{4}{3}(8) - \underbrace{\lim_{t\to 0} \frac{4}{3}t^{3/4}}_{=0} = \frac{32}{3}$$

2. Let $a_n = \frac{\cos^2 n}{n^2}$. Does $\lim_{n\to\infty} a_n$ exist? If it does, find its value.

$$0 \le \left|\frac{\cos^2 n}{n^2}\right| \le \frac{1}{n^2}$$

as $n \to \infty$: $0 \to 0$ and $\frac{1}{n^2} \to 0$

So $\lim_{n\to\infty} \left|\frac{\cos^2 n}{n^2}\right| = 0$

Therefore (discussed in text/class)

$$\lim_{n\to\infty} \frac{\cos^2 n}{n^2} = 0$$

Math 132, Spring 2017
Quiz 6 March 28, 2017
For all 12 p.m. Sections
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<u>Do NOT use any methods except those discussed so far in this course.</u>

1. Does $\int_0^1 \frac{e^x}{e^x - 1}\, dx$ converge or diverge? If it converges, find its value. (*Be sure to include a step that shows the <u>limit</u> used to answer the question.*)

$$\int_0^1 \frac{e^x}{e^x-1}\,dx$$

$$\int \frac{e^x}{e^x-1}\,dx \underset{\substack{u=e^x-1\\du=e^x dx}}{=} \int \frac{du}{u} = \ln|u|+C = \ln|e^x-1|$$

$$\int_0^1 \frac{e^x}{e^x-1}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{e^x}{e^x-1}\,dx = \lim_{t\to 0^+} \ln|e^x-1|\Big|_t^1 = \lim_{t\to 0^+}\Big[\ln(e-1) - \underbrace{\ln(e^t-1)}_{\to -\infty}\Big]$$

so limit d.n.e. ($=\infty$)

2. Let $a_n = \ln(2n^2+1) - \ln(n^2+1)$. Does $\lim\limits_{n\to\infty} a_n$ exist? If it does, find its value.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(\frac{2n^2+1}{n^2+1}\right) = \ln\left(\lim_{n\to\infty}\frac{2n^2+1}{n^2+1}\right)$$

$$= \ln\left(\lim_{n\to\infty}\frac{2+1/n^2}{1+\frac{1}{n^2}}\right) = \ln 2.$$