

Math 132, Spring 2017

Quiz 7 April 11, 2017

For all 8 a.m. Sections

Show enough work to make it clear how you got your answer.

Do NOT use any methods except those discussed so far in this course.

1. Use either the comparison test or the limit comparison test to decide whether $\sum_{n=3}^{\infty} \frac{2n}{3n^3+n}$ converges or diverges.

The limit comparison test is easier: Since $\frac{2n}{3n^3+n} = \frac{2}{3n^2+1} \xrightarrow{n \rightarrow \infty} \frac{2}{3n^2}$,
I suspect the series is comparable to $\sum_{n=3}^{\infty} \frac{1}{n^2}$.

$$\lim_{n \rightarrow \infty} \frac{\frac{2n}{3n^3+n}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{2n}{3n^3+n} \cdot \frac{n^2}{1} = \lim_{n \rightarrow \infty} \frac{2n^3}{3n^3+n} \leq \lim_{n \rightarrow \infty} \frac{2}{3 + \frac{1}{n^2}} = \frac{2}{3} (\neq \infty)$$

By the limit comparison test,
 $\sum_{n=3}^{\infty} \frac{2n}{3n^3+n}$ converges because $\sum_{n=3}^{\infty} \frac{1}{n^2}$ converges.

2. Does the integral test apply to $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$? Does the series converge or diverge?

For $x \geq 2$: ~~$f(x) = \frac{1}{x(\ln x)^2} > 0$~~
 $f(x)$ is decreasing
 $f(x)$ is continuous } so integral test applies.

$$\int_2^{\infty} \frac{1}{x(\ln x)^2} dx ?$$

$$\int \frac{1}{x(\ln x)^2} dx \stackrel{u = \ln x}{=} \int u^{-2} du = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$$

$$\text{So } \int_2^{\infty} \frac{1}{x(\ln x)^2} dx = \lim_{t \rightarrow \infty} \int_2^t \frac{1}{x(\ln x)^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{\ln x} \right]_2^t = \lim_{t \rightarrow \infty} \left(-\frac{1}{\ln t} + \frac{1}{\ln 2} \right) = \frac{1}{\ln 2}.$$

Since the integral converges, so does $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

Math 132, Spring 2017
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 For all 9 a.m. Sections

Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.

1. Use either the comparison test or the limit comparison test to decide whether $\sum_{n=3}^{\infty} \frac{e^n}{e^{2n} + 1}$ converges or diverges.

Comparison Test : $\frac{e^n}{e^{2n} + 1} \leq \frac{e^n}{e^{2n}} = \frac{1}{e^n}$
 Since $\sum_{n=3}^{\infty} \frac{1}{e^n}$ converges (Geo. series, $r = \frac{1}{e} < 1$)
 So $\sum_{n=3}^{\infty} \frac{e^n}{e^{2n} + 1} \leq \sum_{n=3}^{\infty} \frac{1}{e^n}$ and $\sum_{n=3}^{\infty} \frac{e^n}{e^{2n} + 1}$ converges
 (You could also compare to $\sum_{n=3}^{\infty} \frac{1}{e^n}$ using the limit comparison test)

2. The integral test shows that $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges. Call s the sum of the series. What value of n will guarantee that $|s - s_n|$ is smaller than $\frac{1}{200}$?

$$|s - s_n| < R_n < \int_n^{\infty} \frac{1}{x^3} dx = \lim_{t \rightarrow \infty} \left[\frac{-1}{2x^2} \right]_n^t \\ = \lim_{t \rightarrow \infty} \frac{-1}{2t^2} + \frac{1}{2n^2} = \frac{1}{2n^2}$$

$$\text{So } |s - s_n| \text{ will be } < \frac{1}{200} \text{ if } \frac{1}{2n^2} < \frac{1}{200} \\ 2n^2 > 200 \\ n^2 > 100 \\ n > 10$$

So using $n=11$ will be good enough.

Math 132, Spring 2017
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 For all 10 a.m. Sections

Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.

1. Use either the comparison test or the limit comparison test to decide whether $\sum_{n=3}^{\infty} \frac{n}{12n^5 - 7n}$ converges or diverges.

Since $\frac{n}{12n^5 - 7n} = \frac{1}{12n^4 - 7}$, I suspect the series has the same behavior as $\sum_{n=3}^{\infty} \frac{1}{n^4}$.

use limit comparison test:

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{12n^5 - 7n}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{n}{12n^5 - 7n} \cdot n^4 = \lim_{n \rightarrow \infty} \frac{n^5}{12n^5 - 7n} = \lim_{n \rightarrow \infty} \frac{1}{12 - 7/n^4} = \frac{1}{12} \left(\neq 0 \right) \text{, So series converges because } \sum_{n=3}^{\infty} \frac{1}{n^4} \text{ converges}$$

2. Does the integral test apply to $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln 2n}}$? Does the series converge or diverge?

$f(x) = \frac{1}{x\sqrt{\ln 2x}}$ is $\begin{cases} \text{continuous} & \text{on } [2, \infty) \\ \text{positive} & \\ \text{decreasing} & \end{cases}$ so

The integral test applies.

$$\int \frac{1}{x\sqrt{\ln 2x}} dx = \int \frac{1}{\sqrt{u}} du = \cancel{\dots}$$

$$du = \frac{2}{2x} dx = \frac{1}{x} dx$$

$$= 2\sqrt{u} = 2\sqrt{\ln 2x}$$

$$\text{So } \int_2^{\infty} \frac{1}{x\sqrt{\ln 2x}} dx = \lim_{t \rightarrow \infty} \left[2\sqrt{\ln 2x} \right]_2^t = \lim_{t \rightarrow \infty} \left[2\sqrt{\ln 2t} - 2\sqrt{\ln 2} \right] = \infty \text{ d.r.e. } (= \infty)$$

Since the integral diverges,

$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln 2n}}$ also diverges.

Math 132, Spring 2017

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For all 11 a.m. Sections

Show enough work to make it clear how you got your answer.

Do NOT use any methods except those discussed so far in this course.

1. Use either the comparison test or the limit comparison test to decide whether $\sum_{n=3}^{\infty} \frac{\sin^2 n}{2n^2 + n}$ converges or diverges.

Comparison test: $\frac{\sin^2 n}{2n^2 + n} \leq \frac{1}{2n^2 + n} < \frac{1}{2n^2}$

So $\sum_{n=3}^{\infty} \frac{\sin^2 n}{2n^2 + n}$ converges because $\sum_{n=3}^{\infty} \frac{1}{2n^2} = \frac{1}{2} \sum_{n=3}^{\infty} \frac{1}{n^2}$ converges.

2. The integral test shows that $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges. Call s the sum of the series. What value of n will guarantee that $|s - s_n|$ is smaller than $\frac{1}{50}$?

$$S = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$
$$|s - s_n| < R_n < \int_n^{\infty} \frac{1}{x^{3/2}} dx = \lim_{t \rightarrow \infty} \left[\frac{-2}{\sqrt{x}} \right]_n^t$$
$$= \lim_{t \rightarrow \infty} \left(\frac{-2}{\sqrt{t}} + \frac{2}{\sqrt{n}} \right) = \frac{2}{\sqrt{n}}$$

$$\text{so } |s - s_n| < \frac{1}{50}$$

$$\frac{2}{\sqrt{n}} < \frac{1}{50}$$

$$\frac{\sqrt{n}}{2} > 50$$

$$\sqrt{n} > 100$$
$$n > (100)^2 = 10^4$$

So $n = 10001$ is good enough.

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For all 12 p.m. Sections

Show enough work to make it clear how you got your answer.

Do NOT use any methods except those discussed so far in this course.

1. Use either the comparison test or the limit comparison test to decide whether $\sum_{n=3}^{\infty} \frac{2^n}{3^n + n^3}$ converges or diverges.

Since 3^n is much larger than n^3 when n is large, I suspect the series behaves like $\sum_{n=3}^{\infty} \frac{2^n}{3^n}$.

Using the limit comparison test,

$$\lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n + n^3}}{\frac{2^n}{3^n}} = \lim_{n \rightarrow \infty} \frac{2^n}{3^n + n^3} \cdot \frac{3^n}{2^n} = \lim_{n \rightarrow \infty} \frac{3^n}{3^n + n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{1 + n^3/3^n} = 1 \quad (\neq 0, \neq \infty), \text{ so } \sum_{n=3}^{\infty} \frac{2^n}{3^n + n^3} \text{ converges}$$

Since $\sum_{n=3}^{\infty} \left(\frac{2^n}{3^n}\right)$ converges (geo series, $r = \frac{2}{3}$) // or, use a suitable comparison
 $\sum_{n=3}^{\infty} \frac{2^n}{3^n + n^3} < \sum_{n=3}^{\infty} \frac{2^n}{3^n}$

2. Does the integral test apply to $\sum_{n=2}^{\infty} \frac{1}{(2n-1)^4}$? Does the series converge or diverge?

Since $f(x) = \frac{1}{(2x-1)^4}$ is continuous + positive + decreasing on $[2, \infty)$, the

integral test applies.

$$\int \frac{1}{(2x-1)^4} dx = \int \frac{1}{2} \cdot \frac{1}{u^4} du = -\frac{1}{6} u^{-3} + C = -\frac{1}{6(2x-1)^3} + C$$
$$\text{So } \int_2^{\infty} \frac{1}{(2x-1)^4} dx = \lim_{t \rightarrow \infty} -\frac{1}{6(2x-1)^3} \Big|_2^t = \lim_{t \rightarrow \infty} -\frac{1}{6(2t-1)^3} + \frac{1}{6(3^3)}$$
$$= \frac{1}{6(27)}$$

Since the integral converges,

$$\text{so does } \sum_{n=2}^{\infty} \frac{1}{(2n-1)^4}.$$