

Math 132, Spring 2017
 Quiz 9 April 25, 2017
 For all 8 a.m. Sections

Show enough work to make it clear how you got your answer.
Do NOT use any methods except those discussed so far in this course.

1. Write a power series representation for the function $f(x) = \frac{3}{3+x}$. For what x 's does it converge?

$$\frac{3}{3+x} = \frac{3}{3(1+\frac{x}{3})} = \frac{1}{1-(-\frac{x}{3})} = \frac{a}{1-r} \quad \text{where } a=1, r=-\frac{x}{3}$$

$$\frac{3}{3+x} = 1 - \frac{x}{3} + \frac{x^2}{9} - \frac{x^3}{27} + \dots = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^n, \text{ which}$$

converges when $|r| = |-\frac{x}{3}| = |\frac{x}{3}| < 1$
 that is, when $|x| < 3$
 or $-3 < x < 3$

2. The power series $\sum_{n=0}^{\infty} (-1)^n \frac{(2x-4)^n}{n+1}$ has radius of convergence $R = \frac{1}{2}$. What is the interval of convergence?

$$\sum_{n=0}^{\infty} (-1)^n \frac{(2x-4)^n}{n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{(2(x-2))^n}{n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{2^n (x-2)^n}{n+1}$$

Ratio Test: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{2^{n+1} (x-2)^{n+1}}{n+2} \cdot \frac{n+1}{2^n (x-2)^n} \right|$

$$= \lim_{n \rightarrow \infty} \frac{n+1}{n+2} \cdot 2 |x-2| = 2|x-2|$$

Series convs when $2|x-2| < 1$
 $|x-2| < \frac{1}{2}$
 $-\frac{1}{2} < x-2 < \frac{1}{2}$
 $\frac{3}{2} < x < \frac{5}{2}$

when $x = \frac{5}{2}$,
 Series is $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n+1} \cdot \left(\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1}$: convs (alt. harmonic series)

when $x = \frac{3}{2}$,
 Series is $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n+1} \left(\frac{-1}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^{2n} \frac{1}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$: divgs (harmonic series)

So interval of conv. is $\left(\frac{3}{2}, \frac{5}{2}\right]$

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Show enough work to make it clear how you got your answer.
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1. Use a power series representation for $\frac{1}{1+x^2}$ to find a power series representation for the function $\frac{x^2}{1+x^2}$. For what x 's does it converge?

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 - x^2 + x^4 - x^6 + \dots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

for $|x^2| < 1$
 so $|x| < 1$
 or $-1 < x < 1$

$$\text{Then } \frac{x^2}{1+x^2} = x^2 - x^4 + x^6 - x^8 + \dots = \sum_{n=0}^{\infty} (-1)^n x^{2n+2} \quad \text{for } |x| < 1$$

2. Find the radius of convergence R for the power series $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n5^n}$

$$\text{Ratio test: } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)5^{n+1}} \cdot \frac{n5^n}{x^n} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{n}{5(n+1)} \quad |x| = \frac{1}{5}|x|$$

so series conv when $\frac{1}{5}|x| < 1$
 $|x| < 5$ or $-5 < x < 5$

Radius of conv $R=5$ (around center at $x=0$)

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Show enough work to make it clear how you got your answer.
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1. Write a power series representation for the function $f(x) = \frac{2}{4-x}$. For what x 's does it converge?

$$\frac{2}{4-x} = \frac{2}{4(1-\frac{x}{4})} = \frac{\frac{1}{2}}{1-\frac{x}{4}}$$

so $f(x) = \frac{2}{4-x} = \frac{1}{2} + \frac{1}{2}(\frac{x}{4}) + \frac{1}{2}(\frac{x}{4})^2 + \dots = \sum_{n=0}^{\infty} \frac{1}{2} (\frac{x}{4})^n$

which convs when $|r| = |\frac{x}{4}| < 1$
 $|x| < 4$
 or
 $-4 < x < 4$

2. Find the radius of convergence R for the power series $\sum_{n=0}^{\infty} 2^n n^2 (x-1)^n$

Ratio test: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{2^{n+1} (n+1)^2 (x-1)^{n+1}}{2^n n^2 (x-1)^n} \right|$

$$= \lim_{n \rightarrow \infty} 2 \left(\frac{n+1}{n} \right)^2 |x-1|$$

$$= 2|x-1|$$

Series convs when $2|x-1| < 1$
 $|x-1| < 1/2$
 $\frac{1}{2} < x < \frac{3}{2}$

Radius of conv
 $R = 1/2$

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 For all 11 a.m. Sections

Show enough work to make it clear how you got your answer.
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1. Use the power series for $\frac{1}{x+1}$ to find a power series representation for $\ln(x+1)^2$.

$$\frac{1}{x+1} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + \dots = \sum_{n=0}^{\infty} (-1)^n x^n \quad \text{when } |r| = |-x| = |x| < 1, \text{ That is, } -1 < x < 1$$

For those x ,

$$\int \frac{1}{1+x} dx = \ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

$$\Rightarrow 2 \ln|1+x| = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{n+1}}{n+1}$$

$$\ln^2(1+x)$$

2. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{n}$.

Ratio test: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-3)^{n+1}}{n+1} \cdot \frac{n}{(x-3)^n} \right|$

$$= \lim_{n \rightarrow \infty} |x-3| \cdot \left(\frac{n}{n+1} \right) = |x-3|$$

Series conv when $|x-3| < 1$, That is $2 < x < 4$

When $x=2$: Series is $\sum_{n=1}^{\infty} (-1)^n \frac{(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$: diverges (harmonic series)

When $x=4$: Series is $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$: conv (alt. harmonic series)

so interval of convergence is $(2, 4]$

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For all 12 p.m. Sections

Show enough work to make it clear how you got your answer.

Do NOT use any methods except those discussed so far in this course.

1. Use the fact that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x to write a power series that represents the antiderivative $\int e^{x^2} dx$.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x$$
$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$
$$\therefore \int e^{x^2} dx = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!} + C$$

2. Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$

Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\sqrt{n+1} (x+6)^{n+1}}{8^{n+1}} \cdot \frac{8^n}{\sqrt{n} (x+6)^n} \right|$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{8} \left| \frac{n+1}{n} \right| |x+6| = \frac{1}{8} |x+6|$$

Series conv when $\frac{1}{8} |x+6| < 1$
 $|x+6| < 8$

When $x = -14$;
series is

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (-8)^n = \sum_{n=1}^{\infty} (-1)^n \sqrt{n}$$

Diverges since $\lim_{n \rightarrow \infty} a_n \neq 0$

when $x = 2$;
series is

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} \cdot 8^n = \sum_{n=1}^{\infty} \sqrt{n}$$

Diverges since $\lim_{n \rightarrow \infty} a_n \neq 0$

Interval of convergence is $(-14, 2)$