

Background expectations for students in Calculus II (Math 132), Spring 2017

Math 132 (Calculus II) assumes that you have taken Math 131 (Calculus I) or the equivalent at another school. You should already be comfortable with the calculus topics in Chapters 2-5.4 of the Math 132 textbook. This material was covered in Math 131, Fall 2016 (with only a few sections of the textbook omitted).

A list of highlights of the material you should already know would include:

Continuity, the Intermediate Value Theorem, limits, the definition of the derivative, how to differentiate all the “standard” functions, differentiation rules (such as the product, quotient, and chain rules), interpretation of the derivative as a rate of change, the Mean Value Theorem, applications of the derivative in finding maxima and minima; the second derivative, concavity, and inflection points; applications to curve sketching, the definition of the definite integral as a limit of Riemann sums, the Fundamental Theorem of Calculus (Parts I-II) and the use of antiderivatives to evaluate simple integrals and find the areas of regions bound by the graphs of functions.

If you are not already fairly comfortable with all these ideas, then you probably should not be taking Calculus II.

The remainder of this page was provided at the beginning of the fall semester. It describes the background needed for Calculus I, and of course these skills are still necessary in Calculus II. In addition to these skills, you need the concentration to read carefully and interpret what you read for the purpose of solving problems.

In problems that come up in calculus, it's often true that more of the actual work depends on precalculus skills than on calculus itself. You should be able to do these things without using a calculator.

1. Manipulate algebraic expressions, including correct calculations with fractions and using the rules of exponents.

Examples: Write  $\frac{a+c}{c} - \frac{c}{a+c}$  as a single fraction and simplify as much as possible.

Simplify  $(\sqrt[3]{x})^{-\frac{1}{2}} \sqrt[6]{x}$  as much as possible.

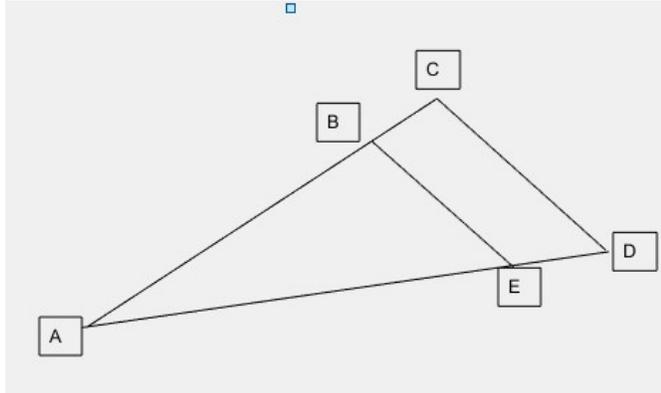
2. Know the basic properties of plane and solid geometric figures – for example, areas of a triangle, parallelogram and circles; area of a sector in a circle; volume of a rectangular box, sphere, cylinder. Know the Pythagorean Theorem and be able to apply it.

Be able to recognize and use proportional relationships derived from similar triangles.

Examples: the points  $P = (0, 0)$ ,  $Q = (1, 1)$ ,  $R = (2, 0)$  and  $S = (3, 1)$  are the

vertices of a parallelogram. What is its area? What is the area of  $\triangle PQS$ ?  
 What is the area and the circumference of the circle that has the line segment  $RS$  as a radius?

In the figure below,  $BE$  and  $CD$  are parallel. If the product of the lengths of  $AB$  and  $CD$  is 5, and the length of  $BE$  is 4, what is the length of  $AC$ ?



4. Determine whether the graphs of two straight lines intersect, and, if so, to find where they intersect; and the same for the graphs of a line and a quadratic function. Be able to find an equation for a line. Be able to determine whether two lines are parallel or perpendicular using their slopes.

Examples: Where do the lines  $3x + 2y = 4$  and  $-x + 4y = 2$  intersect?  
 Are these lines perpendicular?

Where do the lines  $y - 2x + 5 = 0$  and  $2y - 3 = 4x$  intersect?

What is the equation of the straight line through  $(1, 1)$  and perpendicular to the line  $y = \frac{1}{2}x$ ?

5. Solve linear equations and quadratic equations (by factoring, or by using the quadratic formula, when needed). Be able to solve linear and quadratic inequalities.

Examples: The inequality  $1 \leq 5 - 2x \leq 4$  is true for what values of  $x$ ?  
 For what  $x$ 's is the inequality  $x^2 + x - 2 < 0$  true?

Where do the line  $y = 2x$  and the parabola  $y = x^2 + x - 2$  intersect?

6. Be able to complete the square of a quadratic expression and recognize when completion the square is appropriate.

7. Be able to graph quadratic functions, ellipses, circles, hyperbolas.

Examples: Draw the graphs of

- i)  $x^2 - 4x + y^2 + 2y = -2$
- ii)  $9x^2 - 18x + 4y^2 + 8y = 23$
- iii)  $yx - y = 4$

8. Understand the idea of a function and use function notation including the composition of functions. Be able to recognize the functions from which a given function is composed.

Examples: i) For  $f(x) = x^2$ , simplify  $\frac{f(x+h)-f(x)}{h}$  as much as possible.

ii) For what functions  $f, g, h$  is  $f(g(h(x))) = \sin^3(x^2)$   
For the same three functions  $f, g, h$ :

$$\begin{aligned}h(g(f(x))) &= \\g(f(h(x))) &= \\f(h(g(x))) &= \end{aligned}$$

iii) If you have a picture of the graph of  $y = f(x)$ , how would you get a picture of the graph of  $y = f(x + 5)$ ? How would you get a picture of the graph of  $y = -2f(x)$ ?

9. Determine the domain of a function.

10. Be familiar with the graphs of the logarithmic and exponential functions

Examples: i) What is the domain of  $g(x) = \frac{x-1}{(x-1)(x+2)(x^2+x+1)}$ . Is it different from the domain of  $f(x) = \frac{1}{(x+2)(x^2+x+1)}$ ?

ii) What is the domain of  $f(x) = \log_2(2x + 5)$ ?

ii) Draw a reasonable picture of the graph of  $y = g(x) = 2^{(x-2)}$

11. Use the properties of logarithmic or exponential functions to simplify expressions or solve equations.

Examples: i) Simplify  $\log\left(\left(\frac{100}{1000} * 2000\right)^{1/2}\right)$  (Here, "log" means  $\log_{10}$ )  
ii) Using logarithms, solve the equation  $2^{5x+1} = 7$

iii) Decide whether  $\log_3(2x + 1)^2 - \log_3(x^2 + 3) = 2$  has solutions and, if so, find any solutions.

12. Know the definitions of the trigonometric functions, be familiar with their graphs and periodicity, be able to evaluate trigonometric functions using standard triangles, and know the basic trigonometric identities including the Law of Cosines.

- Examples:
- i) What are  $\sin 45^\circ$ ,  $\cos 45^\circ$  and  $\tan 45^\circ$  ?
  - ii) What are  $\sin \theta$  and  $\cos \theta$  if  $\theta = \frac{\pi}{6}$  radians
  - iii) If  $\sin \theta = \frac{1}{3}$ , what is  $\sin (2\theta)$  ?
  - iv) Find  $\sin(75^\circ)$ , using that  $75^\circ = 30^\circ + 45^\circ$
  - iv) If the three sides of a triangle have lengths 2, 3, and 4, then what is the angle between the sides that have lengths 2 and 3? (approximate answer, correct to one decimal place)