Discussion Section, Math 132, Spring 2017, Solutions
February 14, 2017

1. A particle movesalong a straight line. At time t (minutes) its position is $s(t) = 2t + \sin(\frac{\pi t}{2})$.

a) Use the function $s(t)$ to calculate the average velocity of the particle for $1 \leq t \leq 3$.

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}} = \frac{s(3)-s(1)}{3-1} = \frac{(6-1)-(2+1)}{2} = 1 \text{ (km/min)}$$

b) Calculate the average velocity another way using $v(t)$

velocity $= v(t) = \frac{ds}{dt} = 2 + \frac{\pi}{2}\cos(\frac{\pi t}{2})$.
The average value of v over the interval $[1,3] = \frac{1}{3-1}\int_1^3 v(t)\,dt$

$$= \frac{1}{3-1}\int_1^3 2 + \frac{\pi}{2}\cos(\frac{\pi t}{2})\,dt = \frac{1}{2}(2t + \sin(\frac{\pi t}{2}))\Big|_1^3$$
$$= \frac{(6-1)-(2+1)}{2} = 1 \text{ (km.min)}$$

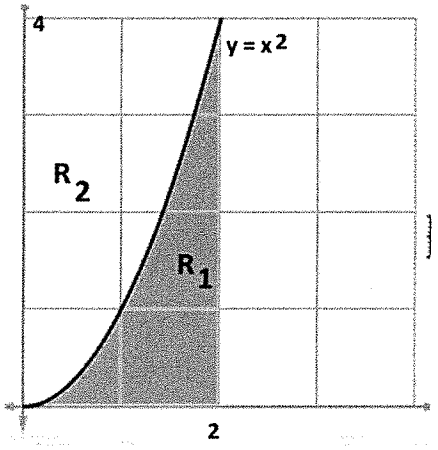
c) Write a formula that shows the general relationship between these two ways of calculating average velocity.

$$v(t) = s'(t)$$

$$\underset{[a.b]}{\text{avg}} \; v = \frac{1}{b-a}\int_a^b v(t)\,dt = \frac{s(b)-s(a)}{b-a}$$

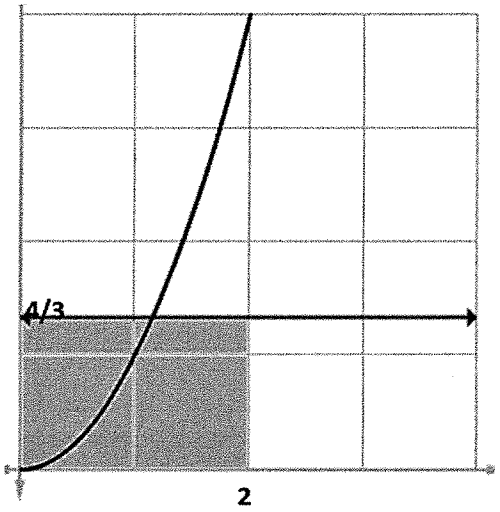
(The two ways of computing "average velociy" are related via the Fundamental Theorem of Calculus, Part II)

2. Consider the following picture:



a) What is the average height of the shaded region R_1 ? Draw a picture to illustrate in terms of areas.

Average height $= \frac{1}{2-0}\int_0^2 x^2\,dx = \frac{1}{2}(\frac{x^3}{3})\Big|_0^2 = \frac{1}{2}(\frac{8}{3}) = \frac{4}{3}$



Area of the shaded rectangle $=$ Area of R_1

b) What is the average width of the unshaded region R_2 ? Draw a picture to illustrate in terms of areas.

Average width of $R_2 = \frac{1}{4}\int_0^4 \sqrt{y}\,dy = \frac{4}{3}$

c) Write an integral that gives the average width of the shaded region R_1.

For $0 \le y \le 4$, the width of $R_1 = w(y) = 2 - \sqrt{y}$.
so the average width is $\frac{1}{4-0}\int_0^4 2 - \sqrt{y}\,dy = \frac{2}{3}$