Math 132, Spring 2017
Discussion Sheet Solutions
April 11, 2017

1. For which of the following series can you determine convergence or divergence by a comparison to $\sum_{n=1}^{\infty} \frac{1}{5n}$?

A) $\sum_{n=1}^{\infty} \frac{1}{5n-3}$ B) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{5n+6}$ C) $\sum_{n=1}^{\infty} \frac{1}{5n+7}$

The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{5n+6}$ cannot be used in a comparison test since its terms are not all positive.

$\frac{1}{5n-3} > \frac{1}{5n}$ and $\sum_{n=1}^{\infty} \frac{1}{5n} = \frac{1}{5}\sum_{n=1}^{\infty} \frac{1}{n}$ which diverges. So the "larger" series $\sum_{n=1}^{\infty} \frac{1}{5n-3}$ must also diverge.

$\frac{1}{5n+7} < \frac{1}{5n}$ so a comparison to the divergent series $\sum_{n=1}^{\infty} \frac{1}{5n}$ $(=\infty)$ does not help us decide whether the "smaller" series $\sum_{n=1}^{\infty} \frac{1}{5n+7}$ converges or diverges

2. Consider the series $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \cdots$

a) Does the series converge or diverge? Why? Can you give two equally good reasons?

The series is an <u>alternating series:</u> $\sum_{n=1}^{\infty} (-1)^{n+1} b_n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2^n}$. Since $b_n = \frac{1}{2^n} > 0$, and $b_n \to 0$, and $b_n > b_{n+1}$, the alternating series test applies and the series converges.

This series is <u>also</u> a geometric series with $a = \frac{1}{2}$ and $r = -\frac{1}{2}$. Since $|r| < 1$, the series converges.

b) What is the sum of the series, s?

For an alternating series, we can't usually determine the sum s. But since this is also a convergent geometric series, we know its sum is $\frac{a}{1-r} = \frac{\frac{1}{2}}{1-(-\frac{1}{2})} = \frac{1}{3}$.

c) In this particular example we could actually find $s = \frac{1}{3}$. But for any alternating series, we can at least approximate s by a partial sum s_n.

In this example, we can write

$$|s - s_{10}| \leq b_{11} = \frac{1}{2^{11}} = \frac{1}{2048} \qquad (*)$$

Does the error estimate (*) turn out to be correct? (*you probably need a calculator*)

For the series given above, $s_{10} = 0.333011612354086$ (*rounded to 15 decimal places*)

and we know that $s = \frac{1}{3}$ so the error estimate works out as predicted:

$$|s - s_{10}| = |\tfrac{1}{3} - 0.333011612354086| = 0.000321720979247708\overline{3}$$

$$< \tfrac{1}{2048} = 0.000488281250000$$

d) In this example, is the approximation s_{10} larger or smaller than s?

Since the alternating series begins with a positive term $b_1 = \frac{1}{2}$, the even partial sums increase to the sum s. So s_{10} is smaller than s and $s - s_{10} > 0$.