Comments on two of the WW9 problems.

3. There's no issue with this problem, but it does take a little more cleverness than most.

The characteristic polynomial is degree 3 (a "cubic" polynomial); k appears as the constant term in the polynomial.

Call the chracteristic polynomial $P(\lambda)$.

Try graphing $y = P(\lambda)$ after setting $k = 0$. You should see a local maximum and local minimum. You will probably want to locate where these occur and calculate the y value at the local maximum and minimum. (*This uses calculus, but very little – at the level of Calc I.)*

Now, if you change k, the effect is just to translate the graph vertically up or down. Your job is to pick the values for k that will make the equation $P(\lambda) = 0$ have 3 roots – that is, you want the graph of $y = P(\lambda)$ to cross the horizontal axis 3 times. It should be clear that this won't happen if k is too big or too small. What k's will make it happen?

16. The problem is not worded well.

V is a plane through $\mathbf{0}$ in $\mathbb{R}^3$. It is a 2 dimensional subspace and you are given a basis for V : $\{\boldsymbol{b_1}, \boldsymbol{b_2}\}$

You're also given a 3×3 matrix A. A is used to define a mapping from the plane V to $\mathbb{R}^3$.

For $\boldsymbol{x}$ in the plane V : $T(\boldsymbol{x}) = A\boldsymbol{x}$ a vector in $\mathbb{R}^3$
We're thinking of the domain of T not as all of $\mathbb{R}^3$
but only those $\boldsymbol{x}$ in the given subspace (plane) V.

It turns out that the image of V $= T(V)$ is a 2-dimensional subspace W of $\mathbb{R}^3$ (another plane through $\mathbf{0}$), and $\boldsymbol{b_1}, \boldsymbol{b_2}$ turn out to be in W. This is because of how WW designed the matrix A.

You could verify all this for your particular A and $\boldsymbol{b_1}$, $\boldsymbol{b_2}$ but it's probably not obvious from just loking at the matrix and vectors., $\boldsymbol{b}$

Therefore $T : V \to W$, and $\mathcal{B} = \{\boldsymbol{b_1}, \boldsymbol{b_2}\}$ is a basis for both V and W.

The matrix for T with respect to $\mathcal{B}$ is a 2×2 matrix and this is the matrix WW wants.