See the introductory example for Chapter 1 (pp. 1-2) and Example 1 in Section 1.6 (pp. 49-51)

Suppose an "economy" has only 4 sectors: agriculture (A), Energy (E), manufacturing (M) and transportation (T). In this simple example, all goods produced are bought and sold among these four sectors – so this model is called a "closed exchange economy."

Each column in following exchange matrix shows how \$1 of goods produced by one sector is consumed by the 4 sectors.

$$\begin{array}{c} \text{Production of sector} \\ \begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ A & E & M & T \end{array} \\ \begin{bmatrix} .65 & .30 & .30 & .20 \\ .10 & .10 & .15 & .10 \\ .25 & .35 & .15 & .30 \\ 0 & .25 & .40 & .40 \end{bmatrix} \end{array} \rightarrow \text{consumed by} \quad \left\{ \begin{array}{l} A \\ E \\ M \\ T \end{array} \right.$$

Going down the third column, for example, we can see how much of each \$1 of goods produced by the manufacturing sector is consumed by the 4 sectors:

0.30 (or 30%) goes to the agriculture sector,
0.15 (or 15%) goes to the energy sector,
0.15 (or 15%) goes to the manufacturing sector
(it uses up some of its own goods), and
0.40 (or 40%) goes to the transportation sector.

Of course, the sum of each column is \$1 (100%).

Going across the second row, for example, we can see how much the energy sector consumes from the others: for example, the energy sector consumes 0.10 $($or $10\%)$ out of each \$1 of goods produced by the agriculture, energy and transportation sectors and 0.15 (or 15%) of each \$1 of goods produced by the manufacturing sector.

Suppose $p_A,\ p_E,\ p_M,\ p_T$ represent the total production of each sector, measured in \$.

Question: is it possible to find values for $p_A,\ p_E,\ p_M,\ p_T$ so that "everybody's happy" – that is, every sector uses its production to pay for what it needs from the other sectors with nothing left over. If so, then the values p_A, p_E, p_M, p_T are called equilibrium prices for the closed exchange economy.

The “cost” to sector A for what it needs is

$$.65P_A + .30P_E + .30P_M + .20P_T$$

and this cost must be “paid for” with P_A (the value of sector A's goods).
So $\quad .65P_A + .30P_E + .30P_M + .20P_T = P_A$

Similarly, we need

$$\begin{aligned} .10P_A + .10P_E + .15P_M + .10P_T &= P_E \\ .25P_A + .35P_E + .15P_M + .30P_T &= P_M \\ .25P_E + .40P_M + .40P_T &= P_T \end{aligned}$$

Rearranging these equations gives the linear system

$$\begin{aligned} -.35P_A + .30P_E + .30P_M + .20P_T &= 0 \\ .10P_A - .90P_E + .15P_M + .10P_T &= 0 \\ .25P_A + .35P_E - .85P_M + .30P_T &= 0 \\ .25P_E + .40P_M - .60P_T &= 0 \end{aligned}$$

The augmented matrix is

$$\begin{bmatrix} -.35 & .30 & .30 & .20 & 0 \\ .10 & -.90 & .15 & .10 & 0 \\ .25 & .35 & -.85 & .30 & 0 \\ 0 & .25 & .40 & -.60 & 0 \end{bmatrix}$$ and its row reduced echelon

form (entries <u>rounded</u> to 2 decimal places for convenience) is

$$\begin{bmatrix} 1 & 0 & 0 & -2.03 & 0 \\ 0 & 1 & 0 & -0.53 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Writing $\approx$ instead of $=$ (since we rounded decimals), we have

$P_A \approx 2.03P_T$
$P_E \approx 0.53P_T$
$P_M \approx 1.17P_T$
P_T free

<u>Therefore</u>: So if a total production P_T ($) for the transportation sector is chosen, then (mathematically) the productions of the other sectors can be set to values that create an equilibrium. Of course, one would want to assign a reasonable and realistic value.