

Final Exam, Math 309, Fall 2015

Exam is in the morning, **Thursday, December 10, 8 a.m. – 10 a.m. in Lab Sciences 300** (same room as was used for Exams 1 and 2). It might be a good idea to pair off with a friend to check that each one is awake and out of bed in time.

The final exam will cover all the material we covered from Chapters 4 – 6 (from textbook, handouts in class or posted online, items mentioned in the lectures)

Because the College asks for a quick turn-around on assigning grades, the final exam will contain only machine graded questions (multiple-choice, true/false, etc.).

---

I have listed some definitions and theorems below that you should know. Of course, the format of the questions will prevent me from asking you actually state a definition, they could still be questions to at least check whether you know the statements and definition: for example

(T/F) The Gram Schmidt Process is a method for converting an orthogonal base for a subspace of  $\mathbb{R}^n$  into an orthonormal base.

(T/F) The inner product of two vectors  $u$  and  $v$  in  $\mathbb{R}^n$  is defined to be the matrix product  $uv^T$

My guess is that final exam scores will show up in BB late Thursday or else Friday morning, and that final grades will be submitted late Saturday, 12/12. I will email the class as soon as I've finishing entering

---

Terms/definitions you should know and things you should be able to do. I tried to list all the most important points, but I could have overlooked something. You are responsible for any of the material we covered in Chapters 4-5. (I think the page numbers are all correct, but, if not, they're close)

- subspace of a vector space (p. 195)
- the null space of a matrix (p. 201) / the kernel of a linear transformation (p. 206)
- the column space of a matrix/the range of a linear transformation (p. 203, 206)
- basis for a subspace of a vector space (p. 211)
- the coordinate vector of  $x$  with respect to a basis (p. 218)
- the change of coordinates matrix (for example, from  $-$ coordinates to standard coordinates or in the opposite direction) (p. 221)
- isomorphism (p. 222)
- the dimension of a vector space (p. 228)
- the row space of a matrix (p. 233)
- the rank of a matrix p. 235
- eigenvector of a square matrix (Notes on Introduction to Diagonalization, or p. 269)

- eigenvalue of a square matrix (Notes on Introduction to Diagonalization, or p. 269)
- the eigenspace of a square matrix corresponding to an eigenvalue (p. 270)
- the characteristic polynomial of a square matrix (p. 276)
- similar matrices (p. 279)
- diagonalizable matrix (Notes on Introduction to Diagonalization, or p. 284)
- define length of a vector and distance between two vectors in terms of inner product (p. 333, 335)
- define orthogonal vectors (p. 336)
- define the orthogonal complement of a subspace of (p. 336)
- state the formula relating the angle between vectors and the inner product (p. 337)
- define the orthogonal projection of a vector  $\mathbf{y}$  onto a vector  $\mathbf{u}$ , and the component of  $\mathbf{y}$  orthogonal to  $\mathbf{u}$  (p. 346); also the projection of  $\mathbf{y}$  onto a subspace  $W$  and the component of  $\mathbf{y}$  orthogonal to  $W$ . (p. 350)
- define an orthogonal matrix (p. 344)
- to state the meaning of “least squares solution” to (p. 362)

You should also understand the content of these theorems and these techniques

- to state the Spanning Set Theorem (p. 210)
- to describe/find bases for  $\text{Nul } A$  and  $\text{col } A$  (pp. 213-214)
- to describe the matrices that you multiply by to change from  $x$ -coordinates to standard coordinates, and from standard coordinates to  $x$ -coordinates (p. 221)
- to state a theorem about expanding a set of vectors to a basis (p. 229)
- to state the Basis Theorem (p. 229)
- to state the Rank Theorem (p. 235) (using rank as defined in terms of the column space)
- to state a theorem about when a square matrix is diagonalizable (Notes on Introduction to Diagonalization, or p. 284)
- to state a theorem about eigenvectors that have different eigenvalues (p. 272)
- to find the eigenvalues for  $A$  in simple cases (the main idea on pp. 277-9)
- to diagonalize a matrix  $A$ , when possible, in simple cases (pp. 283-284)
- write the matrix for a linear transformation from  $V$  to  $W$  with respect to bases  $\mathcal{B}_V$  for  $V$  and  $\mathcal{B}_W$  (p. 291)
- to state the Orthogonal Decomposition Theorem (p. 350) and the Best Approximation Theorem (p. 352)
- to describe and use the Gram Schmidt Process (p. 356-358)
- state a theorem about when a least squares solution is unique (p. 365)
- to define the normal equation for  $Ax = b$
- to write the equations needed to find the “best fitting function” of a certain type to a set of data points (pp. 370-375)
- to do any calculations associated with all of these things