Where we are in relation to the textbook?

In the lectures and notes distributed, we have already covered much of the material in Sections 5.1-5.3. The supplementary materials that you should have read so far are listed below: these were handed out in class and are also linked via the syllabus.

1) Introduction to Diagonalization
(at http://www.math.wustl.edu/~freiwald/309diagonalization.pdf)

2) Diagonalization Example
(at http://www.math.wustl.edu/~freiwald/309diagexample.pdf)

3) Example: A Markov Process
(at http://www.math.wustl.edu/~freiwald/309markov.pdf)

4) You might also find some of the notes in the lecture pdf files useful – although they were put together as "talking points" for lectures rather than as reading material.

http://www.math.wustl.edu/~freiwald/309lect24pdfstudent.pdf
http://www.math.wustl.edu/~freiwald/309lect25pdfstudent.pdf
http://www.math.wustl.edu/~freiwald/309lect27pdfstudent.pdf

Still to cover (in red)

In Section 5.1: we have covered everything except Theorem 2, p 270. That will be included in Friday's lecture.

In Section 5.2: we have covered the "characteristic equation" and how to find eigenvalues (*easy in theory, sometimes very hard in practice*). What we said was that the following are equivalent statements:

λ is an eigenvalue of A	if and only if
$A\boldsymbol{x} = \lambda\boldsymbol{x}$ has nontrivial solutions	if and only if
$(A - \lambda I)\boldsymbol{x} = \mathbf{0}$ has nontrivial solutions	if and only if
$(A - \lambda I)$ is not invertible	if and only if
$\mathbf{det}(\boldsymbol{A} - \boldsymbol{\lambda I}) = 0$	

$\uparrow$

The left side of this equation is the characteristic polynomial of A. Solving this equation (perhaps easier said than done) gives the eigenvalues of A.

In the list above, let $\lambda = 0$: then the following are equivalent (*which I didn't bother to mention in class:* see the middle of p. 275):

$\lambda = 0$ is an eigenvalue if and only if
A is not invertible if and only if
det$(A) = 0$

or, to write these statements in negative form:

$\lambda = 0$ is <u>not</u> an eigenvalue if and only if
A is invertible if and only if
det$(A) \neq 0$

So "$\lambda = 0$ is <u>not</u> an eigenvalue" of A and "det $A \neq 0$" can be added to the list of statements equivalent to "A is invertible" (in the Invertible Matrix Theorem)

The material on determinants in Section 5.2 might be something to review, but the text includes it there for the sake of students whose class might have skipped Chapter 3 $-$ so that material you should already know.

We'll will mention the topic of "similar matrices" (p. 277) in Friday's lecture.

The "Application to Dynamical Systems" on p. 278 is worth reading (*partly to see an example with unpleasant numbers*), but we already covered the idea: it's exactly the same technique that we used in the last two pages of the notes *Example: A Markov Process*.

<u>In Section 5.3</u>

The Diagonalization Theorem (Theorem 5, on p. 282) was already covered as Theorem 1 and Theorem 2 in the notes *Introduction to Diagonalization*.   In those notes I presented the proof, but only for the case of a 2×2 matrix A. I stated that "the same proof" works for $n \times n$ matrices: by now, you should be able to read the $n \times n$ version of the proof in the textbook.

We have already done several examples about diagonalizing matrices (similar to Examples 3 and 4 on p. 283-284). See the notes *Diagonalization Example*, and Monday's notes in class.

We still need to do another example, Theorem 6, and some material related to Theorem 7.