

For learning (and exam) purposes, you should be able to give definitions of the following terms.

Notes: 1) while a paraphrase of a definition is ok, it must be correct. Sometimes students, when trying to paraphrase, actually misstate the definition making it incorrect. The safest things, with definitions, is to memorize them as given.

2) If a definition of a term is asked for, then actually give the definition, NOT some other statement equivalent to the definition.

For example, if asked: the $n \times n$ matrix A is invertible means: _____ then, don't answer "A has n pivot positions." It's true that statement is equivalent to saying A is invertible, but it wasn't the definition of invertibility.

Definitions (note that there is a Glossary in the appendix of the textbook, starting on p. A7, where terms and definition are collected together.)

- 1) What are the elementary row operations (EROs): p. 6
- 2) When is a matrix in an echelon form? (p. 13)
- 3) When is a matrix in reduced row echelon form (rref)? (p. 13)
- 4) What is a pivot column in a matrix A ? (p. 14)
- 5) For a system of linear equations (or a vector equation, or a matrix equation $A\mathbf{x} = \mathbf{b}$), what is a basic variable? what is a free variable? (p. 18)
- 6) If $\mathbf{v}_1, \dots, \mathbf{v}_p$ are in \mathbb{R}^n , what is Span $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ (p. 30)
- 7) If $A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n]$ is an $m \times n$ matrix and \mathbf{x} is in \mathbb{R}^n , then the product of A and \mathbf{x} (written as $A\mathbf{x}$) is defined as: _____ (p. 35)
- 8) Let $S = \{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ be a set of vectors from \mathbb{R}^n . S is linearly independent if _____ (p. 56) State what it means for S to be linearly dependent.
- 9) A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called linear if it satisfies two conditions: _____ and _____ (p. 65)
- 10) A linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called onto if: _____ (p. 75)
- 11) A linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called one-to-one if: _____ (p. 75)
- 12) If A is $m \times n$ and B is $n \times p$, then the product AB is defined to be: _____ (p. 95)

- 13) The row-column rule for computing AB states: _____ (p. 96)
- 14) A linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is invertible means: _____(p. 113)

- 15) An elementary matrix is: _____ (p. 106)
- 16) The transpose of a matrix A is: _____ (p. 99)
- 17) The column-row expansion of AB is: _____ (p. 119)
- 18) in the factorization $A = LU$, describe the matrices L and U (P. 124)

You should be able to answer questions like:

Suppose A is $m \times n$. State 3 conditions, each equivalent to the statement that “the columns of A span \mathbb{R}^n .” (p. 37, *and there are other equivalent conditions later in the Chapter*)

A theorem says that “the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution if and only if: _____ (p. 43)

A theorem says that a linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$, with standard matrix A , is

- i) onto if and only if the columns of A _____
- ii) one-to-one if and only if the columns of A _____ (p. 77)

Suppose A is $n \times n$. Give several statements, each equivalent to the statement that A is invertible. (p. 112) or, in other words, list several of the equivalent statements in the Invertible Matrix Theorem.

Describe the algorithm for deciding whether a square matrix A is invertible and for finding A^{-1} when A is invertible (p. 108)