Other Logical Connectives Used in English

English has many connectives to use for joining propositions together. The simplest are: “not” (\( \sim \)), “and” (\( \land \)), “or” (\( \lor \)), “implies” (\( \Rightarrow \)), “is equivalent to” (\( \iff \)). We wrote down truth tables to see exactly what each connective is intended to mean.

Another connective that we use fairly often in mathematics is “neither/nor.” Here the notation is a little less standard, but \( \downarrow \) is frequently used: \( P \downarrow Q \). This connective \( \downarrow \) is defined to have the same truth table as \( \sim P \land \sim Q \) (or, equivalently, as \( \sim (P \lor Q) \)).

The connective “not both \( P \) and \( Q \)” is also used sometimes. It is written \( P\mid Q \) (or \( P \uparrow Q \) in some books). It has the same truth table as \( \sim (P \land Q) \) (or equivalently, as \( \sim P \lor \sim Q \)).

\[ \text{In this course, we will usually get by with } \sim, \land, \lor, \Rightarrow, \text{ and } \iff. \text{ But, at least, you should also know how to write “neither/nor” in terms of these.} \]

\[ \text{Actually, all possible logical connectives one could invent have an equivalent form that can be written in terms of } \sim, \land, \lor, \Rightarrow, \text{ and } \iff. \text{ This is a theorem from a course in propositional logic. So, in some sense, we know we haven’t short-changed ourselves by using just these.} \]

\[ \text{In fact, we could economize even more if we wanted to. There’s a lot of redundancy in the connectives. If our interest were just to minimize the number of connectives used, we could (for example) completely avoid using } \land: \text{ whenever we want to say “} P \land Q \text{” we could just substitute the equivalent form } \sim (\sim P \lor \sim Q). \text{ But this economy would come at the price of making expressions harder to read.} \]

\[ \text{To take economy to the extreme, it’s possible to express all connectives in terms just one connective: for example, any propositional form using the connectives } \sim, \land, \lor, \Rightarrow, \text{ and } \iff \text{ can be replaced by an equivalent form that uses only “neither/nor” } \downarrow. \]

\[ \text{For example, } \sim P \text{ is equivalent to } P \downarrow P \ (“\text{neither } P \text{ nor } P\”), \text{ and } \]

\[ P \land Q \text{ is equivalent to } (\sim P) \downarrow (\sim Q) \]

\[ \text{Try writing } \lor, \Rightarrow, \iff \text{ in terms of } \downarrow. \]

\[ \text{Certain connectives such as } \downarrow \text{ are special: for example, you cannot write } \sim P \text{ in an equivalent form that uses only } \land. \]
There are other connective words in English that (usually) have the same logical meaning as “and”: these connectives include “but”, “while”, “although.” For example:

John likes spaghetti but John does not like macaroni.

\[ P \quad Q \]

This sentence is true when \( P \) and \( Q \) are true, and false otherwise — so it is logically equivalent to \( P \land Q \).

Although this sentence has the same truth value whether we use “but” or “and”, the use of “but” gives the sentence a different tone. Here “but” suggests an unexpected contrast — because spaghetti and macaroni are such similar foods. In English, someone would probably say “John likes spaghetti and John doesn't like Clint Eastwood” instead of “John likes spaghetti but John doesn't like Clint Eastwood” — even though there's no logical difference.

Sometimes English sentences use connectives in ways that make it unclear how to write the meaning of the connective. This happens when a connective in an English proposition is ambiguous; a decision about the English meaning is needed before you can decide what logical connective is being used.

Example In this example, the meaning of the English connective unless depends on the “story-line.” John and Jane are considering whether to go to Blueberry Hill (BBH) for dinner.

Story 1) John is reluctant to go out. The conversation starts:

John: I won't go to BBH unless BBH has chili.
Jane: BBH has chili.
John: I also won't go to BBH unless BBH has champagne.
Jane: BBH has champagne too.
etc.

Let \[ P = \text{“I will go to BBH”} \]
\[ Q = \text{“BBH has chili”} \]
\[ R = \text{“BBH has champagne”} \]

The connective “unless” in John's first statement means that chili is necessary for going to BBH but chili not sufficient — because John wants other things too, like champagne. In this story,

John's first statement says \( P \Rightarrow Q \) (\( Q \) is necessary for \( P \))
but it doesn't say \( Q \Rightarrow P \) (\( Q \) is sufficient for \( P \))

Story 2) John loves going to BBH and really wants to go there with Jane for their
birthday dinner. His only worry is whether champagne will be available for the celebration: if BBH has it, that's where they'll go for sure.

John says: I won't go to BBH unless BBH has champagne

In the context of this story, “unless” means that champagne is necessary and sufficient to get John to go to BBH. In this context, John is saying \( P \iff R \).

**Exercise:** Try to invention a variation of this story where “unless” means that chili (or champagne) is sufficient but not necessary.