Some Definitions and Notation for Use in Early Proof Examples
(*most of these are in the Preface to the Student in the text, p. xii*)

1) An integer x is <u>even</u> iff we can write $x = 2k$ for some integer k
(Somewhat more formally: $\forall x \in \mathbb{Z}$ (x is even $\Leftrightarrow \exists k \in \mathbb{Z}$ for which $x = 2k$)

2) An integer x is <u>odd</u> iff we can write $x = 2k + 1$ for some integer k
(Somewhat more formally: $\forall x \in \mathbb{Z}$ (x is odd $\Leftrightarrow \exists k \in \mathbb{Z}$ for which $x = 2k + 1$)

3) For natural numbers a and b, <u>a divides b (written a|b)</u> iff $b = ak$ for some natural number k. If $a|b$, then a is called a <u>divisor</u> or <u>factor</u> of b. More formally ,
for natural numbers a and b: $(a|b \Leftrightarrow \exists k \in \mathbb{N}$ for which $b = ak$.
The definition of "divides" for integers is almost identical:
for integers a and b: $(a|b \Leftrightarrow \exists k \in \mathbb{Z}$ for which $b = ak$

4) A natural number p is <u>prime</u> iff $p > 1$ and the only divisors of p are p and 1. More formally,
for a natural number p : p is prime $\Leftrightarrow$ $(p > 1) \wedge (a|p \Rightarrow (a = 1 \vee a = p))$

5) A real number x is called <u>rational</u> iff we can write $x = \frac{p}{q}$ where p and q are integers and $q \neq 0$. More formally,

for a real number x : x is rational $\Leftrightarrow (\exists p \in \mathbb{Z})(\exists q \in \mathbb{Z})$ $(x = \frac{p}{q} \wedge x \neq 0)$

6) For a nonnegative integer n, <u>n factorial</u> (denoted by $n!$) is defined as follows:

$$
\begin{aligned}
0! &= 1 \\
1! &= 1 \\
2! &= 1 \cdot 2 \\
3! &= 1 \cdot 2 \cdot 3 \\
&\vdots \\
n! &= 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n
\end{aligned}
$$

7) If x is a real number, we define

<u>the floor function</u>, $\lfloor x \rfloor =$ the largest integer that is $\leq x$
For example, $\lfloor 3.1 \rfloor = 3$, $\lfloor -4.1 \rfloor = -5$.
The floor function is also called the "greatest integer function."

<u>the ceiling function</u>, $\lceil x \rceil =$ the smallest integer that is $\geq x$
For example, $\lceil 3.1 \rceil = 4$ and $\lceil -4.1 \rceil = -4$.

<u>the absolute value function</u>, $|\, x \,| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$