

Using Parentheses with Logical Connectives

There are some standard rules about how connectives are grouped (“how parentheses should be inserted”) when a propositional form has been written ambiguously. If we all follow these rules, then there really is no ambiguity.

1) Apply the connectives, inserting parentheses if needed, in the following preferential order:

- | | | |
|------|-------------------|---|
| i) | \sim | applies to the shortest proposition to its right |
| ii) | \wedge | applies to the shortest propositions on each side of it |
| iii) | \vee | " " " |
| iv) | \Rightarrow | " " " |
| v) | \Leftrightarrow | " " " |

2) If at any time you are with “repeats” of the same connective, group them by working from left to right: for example, $P \vee Q \vee R$ means $(P \vee Q) \vee R$

Example: insert parentheses according to these rule to make the proposition easier to read

$$S \Leftrightarrow P \wedge Q \wedge \sim R \vee S \Rightarrow T \vee Q \Leftrightarrow R$$

Solution: We insert the parentheses in the order i), ... iv). In each step, use 2) if necessary. TO make reading easier, we may also use {}'s and []'s for ()'s.

$$\begin{aligned} S &\Leftrightarrow P \wedge Q \wedge (\sim R) \vee S \Rightarrow T \vee Q \Leftrightarrow R \\ S &\Leftrightarrow ((P \wedge Q) \wedge (\sim R)) \vee S \Rightarrow T \vee Q \Leftrightarrow R && \text{((2) was used here)} \\ S &\Leftrightarrow [((P \wedge Q) \wedge (\sim R)) \vee S] \Rightarrow (T \vee Q) \Leftrightarrow R \\ S &\Leftrightarrow \{ [((P \wedge Q) \wedge (\sim R)) \vee S] \Rightarrow (T \vee Q) \} \Leftrightarrow R \\ (S &\Leftrightarrow \{ [((P \wedge Q) \wedge (\sim R)) \vee S] \Rightarrow [T \vee Q] \}) \Leftrightarrow R && \text{((2) was used here)} \end{aligned}$$

If everyone follows these rules (which are just a set of agreed-upon conventions), then there is never any ambiguity when parentheses are omitted using logical connectives. But the goal is not to remove all parentheses and leave an untidy a mess for the reader to untangle. Insert as many parentheses in what you write as are needed to make it “easy to read”. Always try to write in a way that makes things as easy as possible for the reader.

On the other hand, sometimes the clutter of too many parentheses also make something hard to read. For example, $((\sim P) \vee Q)$ is perfectly correct, but we should be all capable of remembering that it means the same thing as the simpler $\sim P \vee Q$.

When there's any question about how what you wrote should be grouped, the rules above will be applied.