Practice with Contrapositives and Converses

For each proposition, write the contrapositive and the converse. Which ones are true and which ones are false?

- 1. Suppose f is a function defined on the real numbers.
 - a) If f has a derivative at 0, then f is continuous at 0
 - b) If f'(x) = 0 for every x, then f(x) = 0 for every x.
 - c) If f has a local maximum at x = 0, then f'(x) = 0
- 2. If the equation $ax^2 + bx + c = 0$ has no real roots, then $b^2 4ac \le 0$.

Practice with Quantifiers

Universe U = the set of all people

 $\begin{array}{ll} R(x) & x \text{ is a robber} \\ L(x) & x \text{ is a liar} \\ M(x) & x \text{ is male} \end{array}$

Translate into English:

$$(\forall x \ R(x)) \Rightarrow (\forall x \ L(x))$$

$$(\exists x \ R(x)) \Rightarrow (\forall y \ L(y))$$

$$\forall x (R(x) \land L(x))$$

$$\forall x \ R(x) \Rightarrow \exists y \left(M(y) \land L(y) \right)$$

$$\forall x ((R(x) \land \sim M(x)) \Rightarrow L(x))$$

$$\forall x ((R(x) \land \sim M(x))) \Rightarrow \forall x L(x)$$

Universe U = all people (living or dead)

F(x)	x is female
P(x, y)	x is a parent of y
M(x, y)	x is married to y
S(x,y)	x and y are siblings

Write in logical notation using these predicates, logical connectives and quantifiers:

Everybody has a brother

No siblings are married to each other.

x is the grandmother of y	(not a proposition, but just a statement $G(x, y)$ about x and $y - a$ new predicate defined by us in terms of the old ones)

G(x,y) :

Everybody has exactly two grandmothers (use G(x, y) from the preceding part)

Try your hand at something more complicated like:

1) x and y are cousins

2) everybody has an uncle

3) x is a bastard