## Practice with Contrapositives and Converses

For each proposition, write the contrapositive and the converse. Which ones are true and which ones are false?

1. Suppose $f$ is a function defined on the real numbers.
a) If $f$ has a derivative at 0 , then $f$ is continuous at 0
b) If $f^{\prime}(x)=0$ for every $x$, then $f(x)=0$ for every $x$.
c) If $f$ has a local maximum at $x=0$, then $f^{\prime}(x)=0$
2. If the equation $a x^{2}+b x+c=0$ has no real roots, then $b^{2}-4 a c \leq 0$.

## Practice with Quantifiers

Universe $U=$ the set of all people

$$
\begin{array}{ll}
R(x) & x \text { is a robber } \\
L(x) & x \text { is a liar } \\
M(x) & x \text { is male }
\end{array}
$$

Translate into English:

$$
\begin{aligned}
& (\forall x R(x)) \Rightarrow(\forall x L(x)) \\
& (\exists x R(x)) \Rightarrow(\forall y L(y)) \\
& \forall x(R(x) \wedge L(x))
\end{aligned}
$$

$$
\forall x R(x) \Rightarrow \exists y(M(y) \wedge L(y))
$$

$$
\forall x((R(x) \wedge \sim M(x)) \Rightarrow L(x))
$$

$$
\forall x((R(x) \wedge \sim M(x))) \Rightarrow \forall x L(x)
$$

Universe $U=$ all people (living or dead)

| $F(x)$ | $x$ is female |
| :--- | :---: |
| $P(x, y)$ | $x$ is a parent of $y$ |
| $M(x, y)$ | $x$ is married to $y$ |
| $S(x, y)$ | $x$ and $y$ are siblings |

Write in logical notation using these predicates, logical connectives and quantifiers:

Everybody has a brother

No siblings are married to each other.
$x$ is the grandmother of $y \quad$ (not a proposition, but just a statement $G(x, y)$ about $x$ and $y$ - a new predicate defined by us in terms of the old ones)

$$
G(x, y):
$$

Everybody has exactly two grandmothers (use $G(x, y)$ from the preceding part)

Try your hand at something more complicated like:

1) $x$ and $y$ are cousins
2) everybody has an uncle
3) $x$ is a bastard
