Errors in the Text and Other Comments

When I used this textbook a few years ago, there were a lot of errors - some mere typos, some more serious. You have a new printing of the text that (supposedly) has eliminated a lot of the mistakes. I have noticed already corrections for several items in Sections 1.1-1.4.

This page will be a collection of typographical errors, mathematical mistakes (I hope, none), confusing sentences, etc. that I notice as the semester goes by. I'll send the list to the publisher when the semester is over.

<u>Please call anything you think is a typo or error to my attention. I'll take a look and, if</u> you're right, I'll add it to this page for everyone's benefit. (*Also tell me if I make a* <u>mistake! Thanks.</u>)

Some of the comments below are worded to address the publisher, not the students in the class.

Chapter 1

<u>p. 2</u> (first sentence, next to last paragraph)

The text says "Statement (h) is an example of a sentence that is neither true nor false, and is referred to as a paradox."

Saying "Statement (h) is an example of a sentence that is neither true nor false" is correct, and "Statement h) is referred to as a paradox" is also correct.

If we interpret the text's use of "and" strictly, there's no problem here. But the text's phrasing might be confusing. It might give the impression that "a statement that is neither true or false" is referred to as a paradox - as if the text read "and such statements are referred to as paradoxes".

After all, a student might note that the sentence "Where is my cheese?", at the top of p. 2, is also neither true nor false and ask whether it is also referred to as a paradox.

A bit of clarification would be helpful. Perhaps something like: roughly, a <u>paradox</u> refers to a sentence which <u>appears</u> that it should have a truth value (because of its grammatical form) but for which assigning either possible truth value ("T" or "F") leads to a contradiction.

p. 25 (middle of the first paragraph, which begins "Theorem 1.3 is helpful...)

Text says "By Theorem 1.2(d)" when it means "By Theorem 1.2(e)"

p. 39 Problem 9a)

The problem asks the student "Where in the proof is it essential that x and y be positive?"

In fact it is not essential; it is sufficient that x and y be <u>nonnegative</u>,

<u>p. 41</u> (preceding the last example on p. 41)

The statement of the Fundamental Theorem of Arithmetic is incorrect:

i) 1 cannot be written as a product of primes.

ii) Can 2 be written as a <u>product</u> of primes? Not unless we agree that a single number, like 2, is considered to be a product with only one factor (itself). This may be a reasonable "convention" to adopt. But, strictly speaking, multiplication involves two "inputs", () × (), so a "product with only one factor" doesn't make sense. Either way, are students supposed to automatically recognize such a convention? iii) The example $6 = 2 \cdot 3 = 3 \cdot 2$ contradicts the uniqueness part of the theorem as stated – unless students are supposed to read into the text an additional meaning – "uniquely except for the order of the factors."

The statement of the theorem should read:

Every natural number <u>larger than 1</u> is <u>either a prime or</u> can be expressed uniquely as a product of primes (<u>except for the order of the factors</u>).

<u>p. 51</u> I think it would be useful for the student if the text pointed out that the "incorrect deductions" 2), 3), 4) are the converses of the "valid deductions" 3), 4), 6).

p. 55 Problem 1j)

There's not really an error. But notice that there are various "trivial" solutions that the text should rule out. For example, let L = G = 0, or let L = 37 and G = 36." Then for any x, L < x < G is false, so $(L < x < G) \Rightarrow (40 > 10 - 2x > 12)$ is true, and therefore $(\forall x)(L < x < G) \Rightarrow (40 > 10 - 2x > 12)$ is true.

The problem should be reworded to say: "Show that there exist integers L and G where L < G such that, for every real number x, ..."

<u>p. 61</u> Last line: text has d = GCD(ab) for d = GCD(a, b)

pp. 62-63 It seems odd to have a big, blue boldfaced "title"

The Division Algorithm For Natural Numbers

as if a new section of the text were beginning. The sinze/color of the font should not give the division algorithm more prominence than Euclid's Algorithm – which certainly is the central topic in pp. 62-62.

- <u>p. 64</u> Just a suggestion: my experience is that if I simply ask a student to find gcd(a, b), the student is likely to avoid the "less familiar" Euclid's Algorithm and instead to construct the gcd by looking for common factors in the prime factorizations of a and b. It's worth pointing out that this is possible, but then saying something like "every computer scientist would tell you that Euclid's Algorithm is the better way: for large a, b finding prime factors can be very time-consuming much more so than the execution of the repeated divisions called for by Euclid's Algorithm."
- <u>p. 65</u> Exercise 10. The hypothesis "q > 1" is also needed.
- <u>p. 66</u> Exercise 11g). "Euclid's Lemma" means "Lemma 1.5" (but the text didn't give it that name)
- p. 75 First sentence of the last paragraph:

"...in proving that $X \subseteq Y$, we did not assume..."

Where did we recently prove " $X \subseteq Y$ "? If the paragraph is generic advice about how to prove $X \subseteq Y$, it would be better to say "..., we do not assume...

Or, was " $X \subseteq Y$ " supposed to be " $A \subseteq B$ "?

p. 75 Second sentence of last paragraph:

"We began by supposing..., and then use ..."

The verbs should be in the same tense.

Chapter 2

<u>p. 79</u> Just a typo in the definition: "Two sets A and B are disjoint..."

<u>p. 88</u> In the proof of Theorem 2.8c), the reference to Exercise 3(b) should, instead, refer to Exercise 4(a) (and maybe also to 4(b))

p. 95 In Exercise 18:

The wording of phrase beginning "<u>That is</u>, suppose..." suggests that you are stating the meaning (definition) "nested."

But what's given is a definition for a <u>decreasing nested</u> sequence of sets. The

"nested" also applies to a sequence where $A_1 \subseteq A_2 \subseteq ...$

(*Also*, "*That is*" *is an awkward way to begin a sentence. Better would be* "... *family of sets – that is, suppose ...*".)

<u>p. 98</u> Next to last line: "...natural number such that $n \in \mathbb{N}$..." should read "...natural number such that $n \in S$..."

<u>**p.** 107</u> Problem 8r) should read $\prod_{i=1}^{n} (1 - \frac{1}{i+1}) = \frac{1}{n+1}$ <u>**not**</u> $\prod_{i=1}^{n+1} (1 - \frac{1}{i+1}) = \frac{1}{n+1}$

Chapter 3

term

<u>p. 143</u> In Exercise 4, p. 143, the headers should read: "Give an example of sets A,B,C, and <u>D</u> such that...

<u>p. 148</u> The text defines, on \mathbb{Z} , a relation \equiv_m for all <u>integers</u> $m \neq 0$. Since m|(x-y) iff -m|(x-y), one only needs to make the definition for m > 0. In fact, on p. 150, the text is proving something about \equiv_m and <u>do</u> assume that m > 0: for example, in lines 10-11 there is a reference to "*m* distinct equivalence classes" for \equiv_m , and, in line 14, we have $0 \leq r \leq m - 1$ (so $m \geq 1$)

The definition of $\equiv m$ should simply be "for every fixed natural number m..."

Chapter 4

p. 181, line 2 ff.

Based on my experiences the last time I taught the course:

A few students seemed to get the impression that the "vertical line test" is some magic rule used IN GENERAL to check whether a function $f : A \rightarrow B$ is one-to-one. This

surprised me. The "vertical line test" only makes sense when when $A, B \subseteq \mathbb{R}$, or, say when $A \subseteq \mathbb{R}^2$ and $B \subseteq \mathbb{R}$. The point is that to think of using the vertical line test, you need a situation where you can picture the graph of f and think about a "vertical lines."

Similar comments apply to the "horizontal line test" (Sect. 4.3, p. 199); but on p. 199, it IS stated that $A, B \subseteq \mathbb{R}$. But then on p. 201, the "horizontal line test for one-to-one" is flat-out called "a visual check of the graph of a function ..." stated without any restriction at all on the sets A, B!!

<u>p. 181, line 8</u> Not really an error, but "...a given relation r..." was probably meant to be "...a given relation R..."

<u>p. 182, first example</u> "...and the set of pre-images of 2 is $\{\frac{\pi}{6} + 2\pi : k \in \mathbb{Z}\} \cup ...$ " Change 2π to $2k\pi$.

p. 184, paragraph containing definition of the canonical map

Typo: "... to a/R, the equivalence class of a, is call the **canonical**..." Change "call" to "called" <u>p. 197, Problem 15b,c</u>) There should be an added assumption that $range(g) \subseteq dom(f)$ so that f(g(x)) is defined on the interval I.

p. 211, Problem 4a)

Answer in the back of the textbook is wrong. The correct answer is: \emptyset

p. 216, Proof of Theorem 4.17 In the 5th line of the proof,

"...implies $|x_n - L| \le \epsilon$..." should be "...implies $|x_n - L| < \epsilon$..."

and also

"...implies $|x_2 - M| < \epsilon$ " should be "..."...implies $|x_n - M| < \epsilon$ "

<u>p. 217, line 5</u>: not really an error, but I think the word "guessing" would be better than "concluding". It turns out that the guess (supported by some calculated data) is correct, but it's not as if it's the conclusion of any airtight argument.

<u>p. 217, line 13:</u> $12n^2$ should be $12n^4$

<u>p. 218, line 5</u> should begin " $1 - L \ge 1$ ", not "1 - L > 1"

<u>Index, p. 368</u> The index entry for "Fundamental Theorem of Algebra" refers to p. 41 – but there is no Fundamental Theorem of Algebra on p. 41. *The error is there because, in the version of the book I used in spring 2006, the "Fundamental Theorem of Arithmetic", on p. 41, was mistakenly called the "Fundamental Theorem of Algebra." When that error was corrected, the index wasn't updated.*