Often we want to take the union of more than two sets, perhaps even infinitely many. We need notation for this. The important things here are to understand 1) what the union of a (possibly infinite) collection of sets means, and 2) that there are several standard ways of writing such unions, all meaning the same thing, Sometimes one notation is more convenient than another. (*All the same comments will also apply to intersections.*)

Definition Suppose $\mathcal{A}$ is a collection of sets. Then

$$\bigcup\mathcal{A} = \{x : x \text{ is in some set in the collection } \mathcal{A}\}$$
$$= \{x : (\exists A \in \mathcal{A}) x \in A\}$$

$\bigcup\mathcal{A}$ is also written as $\bigcup_{A\in\mathcal{A}} A$.

Examples

1. $\mathcal{A} = \{B, C\}$ $\qquad \bigcup\mathcal{A} = \{x : x \in B \text{ or } x \in C\} = B \cup C$

2. $\mathcal{A} = \{A_1, A_2, A_3\}$ $\qquad \bigcup\mathcal{A} = \{x : x \in A_1 \text{ or } x \in A_2 \text{ or } x \in A_3\}$
   $= \{A_i : i \in \{1, 2, 3\}\}$ $\qquad = A_1 \cup A_2 \cup A_3$

3. $\mathcal{A} = \{I_1, I_2, ..., I_n, ...\}$ $\qquad \bigcup\mathcal{A} = I_1 \cup I_2 \cup ... \cup I_n \cup ...$
   $= \{I_n : n \in \mathbb{N}\}$ $\qquad = \bigcup_{n=1}^{\infty} I_n$

   If $I_n = [0, 1 - \frac{1}{n}] = \{x \in \mathbb{R} : 0 \leq x \leq 1 - \frac{1}{n}\}$
   then

   $\bigcup\mathcal{A} = \bigcup_{n\in\mathbb{N}} I_n = I_1 \cup I_2 \cup ... \cup I_n \cup ...$
   $= \bigcup_{n=1}^{\infty} I_n = \bigcup_{n=1}^{\infty} [0, 1 - \frac{1}{n}]$ all mean the same thing

   $=$???

4. Suppose, for each nonnegative real α, $I_\alpha =$ interval $(0, \alpha)$ in $\mathbb{R}$.

   Let $\mathcal{A} = \{I_\alpha : \alpha \in \mathbb{R}^+\}$
   $\uparrow$
   set of nonnegative reals $= \Delta =$ "the indexing set"

   i) $\bigcup\mathcal{A} = \bigcup_{\alpha\in\mathbb{R}^+} I_\alpha = \bigcup_{\alpha\in\Delta} I_\alpha =$???

   ii) What is $\bigcup_{\alpha\in\Delta} I_\alpha$ if each $I_\alpha = \{2^x : x \geq \alpha\}$?

Examples

1. Let $\mathcal{A} = \{I_n : n \in \mathbb{N}\}$, where $I_n = \{x \in \mathbb{R} : 0 \leq x \leq 1 - \frac{1}{n}\} = [0, 1 - \frac{1}{n}]$.

$\uparrow$

$\mathbb{N} = \Delta =$ the indexing set

i) What is $\bigcap \mathcal{A} = \bigcap_{n=1}^{\infty} A_n$?

ii) What is $\mathbb{R} - \bigcap \mathcal{A} = \mathbb{R} - \bigcap_{n=1}^{\infty} A_n$?

2. If B is any set, what are

i) $\bigcup \mathcal{P}(B) =$

ii) $\bigcap \mathcal{P}(B) =$