Some Writing Tips

Ambiguity is permitted for certain kinds of writing - for example, poetry, or a passage in a mystery novel. The ambiguity may be intentional (to puzzle or tease the reader) or it may occur because the subject (a deep emotion, a beautiful scene) is difficult to capture in mere words. Sometimes, however, ambiguity is just the result of bad writing.

Most expository writing, especially in mathematics, wants to avoid ambiguity. It wants to express an observation, idea, argument, or a computation as precisely and clearly as possible. It also tries to be as brief as it can and still do the job effectively. (*The detail and explanation required depend partly on the audience: are the readers going to be other college students? high school students? the general public? research mathematicians?*)

No matter how correctly something is written, communication will not happen unless someone reads it. Therefore another goal of good expository writing is a style that is inviting to the reader. What you write should flow smoothly and should not annoy the reader with unnecessary mental calisthenics to make corrections, to fill your logical gaps, or to find your verb.

It takes time and practice to develop a good writing style, and perhaps this comes easier for some than others. But even the best writers will you that one of the secrets is "revise, revise, revise..." The English writer Samuel Johnson once said "that which is written without effort is usually read without pleasure."

Mathematicians often write long research papers and books. Compared to projects like that, writing solutions for homework exercises is much more manageable. You should view the written homework assignments not just as "problems to solve" but as a setting in which to practice writing mathematics well. Here are some miscellaneous hints, in no particular order, to get started.

1) Start with a rough draft. (*In mathematics, a "first draft" often means your "scratchwork.*") Then think about what you have. Discard all false starts and irrelevant things that you wrote, and rearrange the essentials into a logical order. Play with the solution until the whole argument is crystal clear in your own mind. You can't say something well until it's clear in your own mind what you need to say.

2) Try to write down the argument in a way that makes it as clear on paper as it is in your own mind. Say exactly what you mean – no more, no less. For your homework, write as if your reader is another student in Math 310. (*Do I need to include this detail? Ask yourself what details the average Math 310 student should need to see written down and what should be "obvious."*)

Once a solution is written, put it aside for a while and then come back and reread it. Can you now say something more clearly or directly? Polish your language.

3) In all writing you need to be careful about using pronouns. This is especially true in writing mathematics because careless pronouns lead to ambiguity.

<u>Bad</u>: "Bob and Tom had a hamburger at McDonald's and he got sick." (*Who got sick?*)

<u>Bad</u>: "... the number c can be written as $c = p(a^2 + b^2)$. It can't be factored, so ..."

(<u>What</u> can't be factored? Does "it" refer to $c ? p ? a^2 + b^2$? Perhaps the reader can figure out what "it" refers to, but he or she will be annoyed at being forced to stop and clean up your careless writing.)

<u>Good</u>: "... the number c can be written as $z = p(a^2 + b^2)$. Since p can't be factored, ... "

If a pronoun causes any ambiguity, then either replace it altogether (as I replaced "it" in the preceding example), or somehow rearrange the sentence to remove the ambiguity.

4) It's generally considered bad mathematical style to start a sentence with a symbol.

<u>Bad</u>: "... the number c can be written as $c = p(a^2 + b^2)$. p can't be factored, so ..." <u>Good</u>: "... the number c can be written as $c = p(a^2 + b^2)$. Since p can't be factored, ..."

5) Don't break a mathematical <u>expression</u> (such as $x^3 + 3x^2 + 2x + 5$) at the end of a line. If you must break an <u>equation</u> or <u>inequality</u> at the end of a line, the standard in mathematical writing is to put the connecting symbol (such as " = " or " < ") on the following line.

<u>No</u>	", then $x^3 + 3x^2 + 2x + 5$, so"
<u>No</u>	", then $x^3 + 3x^2 + 2x + 5$, so"
<u>No</u>	", then $x^3 + 3x^2 + 2x + 5 = 3$, so"
<u>If ne</u>	eccessary ", then $x^3 + 3x^2 + 2x + 5$ = 3, so"
Best	if possible ", then

$$x^3 + 3x^2 + 2x + 5 = 3$$
, so ..."

5) Do not annoy the reader with nonstandard or "cute" notation. Use the standard notation or, if you need to create notation, try to do so in a way that helps the reader.

Bad: No one wants to read a page where you begin by saying

"Consider a triangle with angles #, \$, and \aleph , and call the sides opposite these angles %, @, and x. Then $\%^2 = @^2 + x^2 - 2@x \cos \#$ "

There's nothing mathematically <u>wrong</u> with this statement of the Law of Cosines, but you should try to choose names and notation that make things as easy as possible for the reader.

Good:

"Consider a triangle with angles A,B, and C, and call the sides opposite these angles a, b and c. Then $c^2 = a^2 + b^2 - 2ab \cos C$."

Here, upper case letters denote one kind of thing (angles) and lower case letters represent another kind of thing (sides). Moreover, there is a built-in memory aid for the reader: side a is opposite angle A, etc. If a new angle comes up later in the paragraph, the reader has every right to expect that it will be given an upper case name, like D.

Usually in an undergraduate course, the textbook sets the "standard notation" to be used. We hope the author does a good job of it.

6) Writing mathematics involves using mathematical symbols (like $\int_a^b f(t) dt$) and logical symbols (like \Rightarrow). For each of these symbols, there is a standard way of reading it aloud:

$\int_a^b f(t) dt$:	"the integral of f from a to b "	(or something similar)
\Rightarrow :	"implies"	(or something similar)
= :	"equals", or "is", or "is equal to"	(or something similar)

When a piece of written mathematics is read aloud exactly as it's written on paper, the result should be a proper English sentence: for example, there should be a subject and a verb.

Bad:
$$\int_0^x t^2 dt \quad x = 2 \quad \frac{2^3}{3} = \frac{8}{3}$$

(If read aloud, where's the sentence? This is just a collection of statements written down side-by-side, and the reader is left to connect them.)

<u>Good</u>: If x = 2, then $\int_0^x t^2 dt = \frac{2^3}{3} = \frac{8}{3}$

(*Read aloud, this is a perfectly good English sentence: "If x equals 2, then the integral of t*² *from 0 to x is* $\frac{2^3}{3}$ *which is* $\frac{8}{3}$ *."*)

7) <u>A convention</u>: mathematical writing is almost always done in the "first person plural", <u>not</u> "first person singular." "First person plural" involves the reader with the writer.

<u>Bad</u>	Proof I first consider the case where $x > 0$. Then	
Good	Proof We first consider the case where $x > 0$. Then	
	or	
	Proof Let us first consider the case where $x > 0$. Then	
	or (with no "person" at all)	

Proof First consider the case where x > 0. Then...

Except in a very few cases - for example, when the writer is a famous, eccentric old geezer whose in-your-face personality is accepted and enjoyed - the first person singular is considered rather self-centered and arrogant.

8) With regard to "proper English usage," try to practice the things described below so ironically by William Safire. (*I think one or two of them are arguable (see the footnotes), and, as usual, almost every rule has exceptions.*)

The passive voice should never be used.

Do not put statements in the negative form.

Verbs have to agree with their subjects.

Proofread carefully to see if you words out.

If you reread your work, you can find on rereading a great deal of repetition that can be avoided by rereading and editing.

A writer must not shift your point of view.

Don't overuse exclamation marks!!

Place pronouns as close as possible, especially in long sentences, as of 10 or more words, to their antecedents.

Writing carefully, dangling participles must be avoided.

Example:

<u>Bad</u>: Let $f(x) = x^2$. Differentiating, f'(x) = 2x. (What does the participle "differentiating" modify?)

<u>Good</u>: Let $f(x) = x^2$. Differentiating, we get f'(x) = 2x. Let $f(x) = x^2$. Differentiating gives f'(x) = 2x(here, "differentiating" is used as a noun (gerund) not as an adjective (participle).)

If any word is improper at the end of a sentence, a linking verb is.

Take the bull by the hand and avoid mixing metaphors.

Avoid trendy locutions that sound flaky.

Everyone should be careful to use a singular pronoun with singular nouns in their writing.

Always pick on the correct idiom.

Last but not least, avoid clichés like the plague; seek viable alternatives.

And don't start a sentence with a conjunction. (Remember, too, a preposition is a terrible word to end a sentence with¹.)

Remember to never split an infinitive².

The following is not part of the course. It's only for those who enjoy grammar.

¹ On the contrary, to do so is not even newfangled. In 1926, the "grammarian's grammarian" H.W. Fowler (<u>A Dictionary of Modern English Usage</u>) wrote

It was once a cherished superstition that prepositions must be kept true to their name and placed before the word they govern in spite of the incurable English instinct for putting them late... The fact is that... even now immense pains are sometimes expended in changing spontaneous into artificial English... Those who lay down the universal principle that final prepositions are 'inelegant' are unconsciously trying to deprive the English language of a valuable idiomatic resource, which has been used freely by all our greatest writers except those whose instinct for English idiom has been overpowered by notions of correctness derived from Latin standards. The legitimacy of the prepositional ending in literary English must be uncompromisingly maintained....

(473-474)

² On the split infinitive, Fowler writes:

"We maintain... that a real split infinitive though not desirable in itself, is preferable to either of two things, to real ambiguity, and to patent artificiality.

For the first, we will rather write 'Our object is to further cement trade relations' than, by correcting into 'Our object is further to cement ...', leave it doubtful whether an additional object or additional cementing is the point.

And for the second, we take it that such reminders of a tyrannous convention as 'in not combining to forbid flatly hostilities' are far more abnormal than the abnormality they evade.

We will split infinitives sooner than be ambiguous or artificial; more than that, we will freely admit that sufficient recasting will get rid of any split infinitive without involving either of those faults, and yet reserve to ourselves the right of deciding in each case whether recasting is worth while."