Math 417-418 2009-2010 Introduction to Topology and Modern Analysis I & II

We can divide mathematics into a few very broad areas, for example: analysis, algebra, discrete mathematics, topology, geometry, and foundations of mathematics. Some would disagree with this classification and might add, say, probability theory while others would just view probability theory as partly analysis (continuous random variables) and partly discrete mathematics (discrete random variables).

Chopping mathematics into such artificial classifications is certainly not important: the boundaries are very fuzzy and these areas all interact with each other. But almost everyone would agree that if mathematics is divided into a small number of major "pieces," then one of those pieces is topology. So nobody would disagree that topology constitutes a <u>very</u> big piece of mathematics. For example, you may find topologists who call themselves general topologists, algebraic topologists, differential topologists, low-dimensional topologists, geometric topologists, piecewise linear topologists, combinatorial topologists,

Math 417 (and 418 in the spring) will be devoted primarily to general topology: that is where topology began, historically, as a distinct field of mathematics. As the name suggests, general topology consists of the broad foundational material for the whole subject. Not only does it contain some beautiful ideas and theorems, but it is also absolutely essential for further work in topology and in other fields like functional analysis. Near the end of the second semester (Math 418) we might spend a few weeks introducing a little algebraic topology: that will depend partly on the class and on how quickly other topics move along.

As a field, general topology (*once called by a more old-fashioned name, "point-set topology*") is no longer very "research-active" – by now it is a well-defined and time-tested body of results. Most of the interesting questions seem to have been settled. Some research does still continue, but most of it is closely related to set theory, mathematical logic and foundations of mathematics: for example, it has turned out that a long-standing classical problem in general topology called the "normal Moore space conjecture" is unprovable from the standard axioms ZFC for set theory. The important topology research today is in those other sub-fields like algebraic topology, differential topology,

However, a good foundation in general topology is essential to be able to work in those areas.

Faculty members have different philosophies about our Math 417-418 sequence. Some make Math 418 substantially into a course on algebraic topology. Although that's an exciting topic, I have two problems with this:

• Some knowledge of algebra, especially group theory and some ring

theory, is necessary to do algebraic topology: the subject is all about

using

tools from algebra to attack topological questions. But an algebra course like our Math 430 is not a prerequisite for Math 418, and having it as a prerequisite would make the course less accessible for students who are primarily interested in topology as a tool for analysis and who have no particular interest in algebra. Therefore such a 418 course needs to devote a lot of time to teaching algebra, often in a hurried way that doesn't do justice to the algebra.

• Devoting Math 418 to algebraic topology also means that material from general topology is either very hurried in Math 417 or that beautiful, and useful classical results are omitted. My personal feeling is that rushing to get to algebraic topology leads to a sense of anxiety at places where a good command of general topology is really needed. This makes algebraic topology more mysterious and less enjoyable. It's important to take the time to cover general topology carefully and to allow for time and practice to really "digest" the ideas.

The course syllabus for Math 417 will discuss details about homework, exams and grading. It will be posted online by the time classes start. For now, I just want you to know that the <u>textbook</u> for the course is one that I have written. It will be photocopied two-sided, on punched paper and distributed by installments in class (since I continuing revising and correcting each time I use it). I recommend that you get a three-ring binder to hold these pages. There will be about 100-125 sheets (so, 200-250 printed sides) each semester.

There will be a charge of \$10.00 each semester to cover the cost of paper, toner and copying time. You can pay this charge to the secretary in the Mathematics Department Office (Cupples I, room 100). She will give me a list of the people who have paid. The office will accept a check made out to "Washington University Department of Mathematics" or cash (but the exact cash amount is required; the Office cannot "make change"). I will distribute the first 20 pages or so free of charge so that there's no rush; but please try to make your payment by Friday, September 4.

Just so you know what the course is about, I've included a Table of Contents for the material I want to cover in the first semester (Math 417).

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