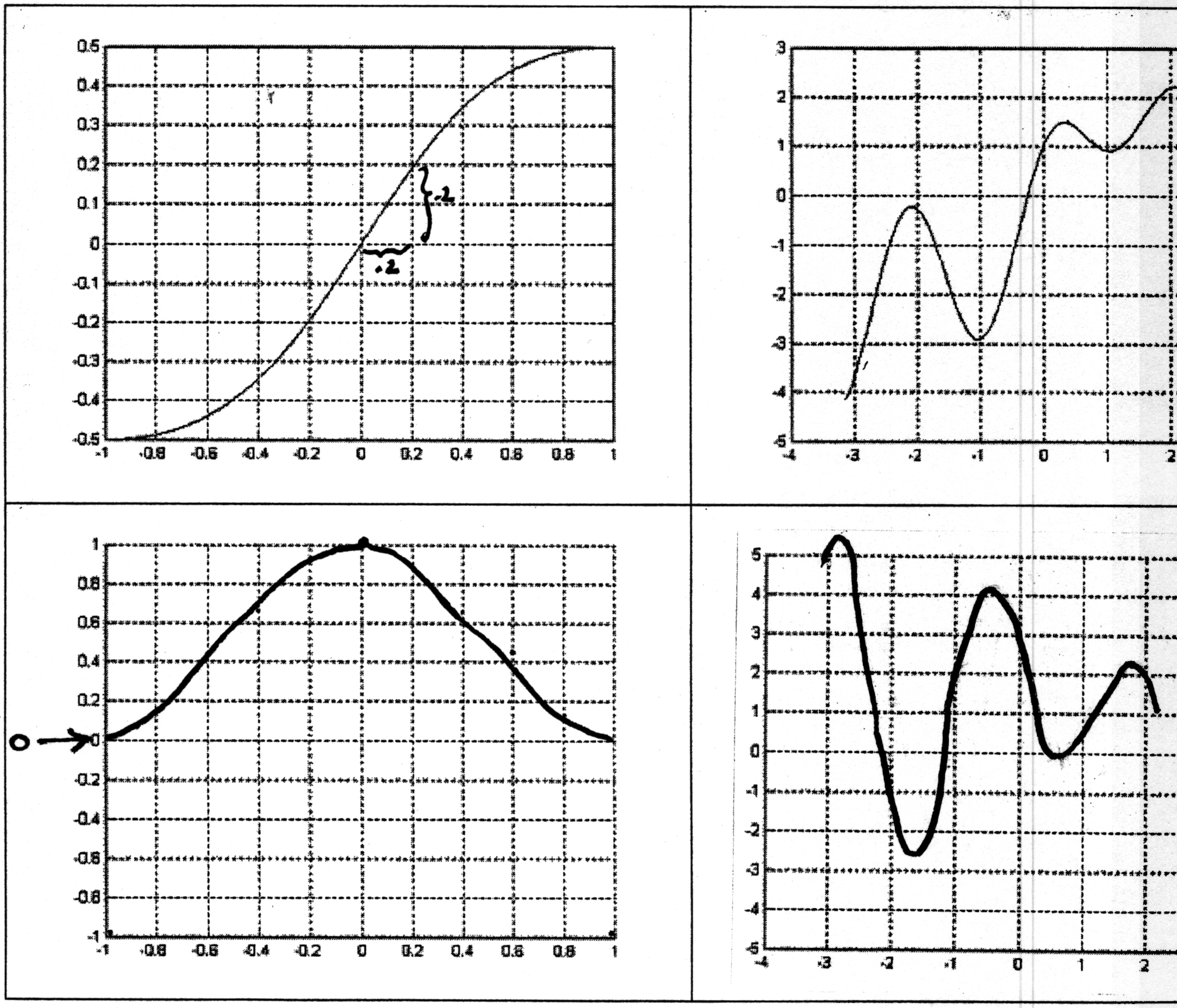
Math 131, Fall 2004
Discussion Section 4

1. The graph of two functions $y = f(x)$ are given in the top row. On the grid below, make a sketch of the graph of $y = f'(x)$ in each case.



2. Each limit represents $\frac{dy}{dx}|_{x=a}$ for some function $y = f(x)$ and some number a. What are f and a?

i) $\lim\limits_{h \to 0} \frac{\cos(h+\pi)+1}{h}$ ii) $\lim\limits_{x \to 1} \frac{x^4+x-2}{x-1}$

Solution: There is no "method" for doing these. The idea is just to remember that

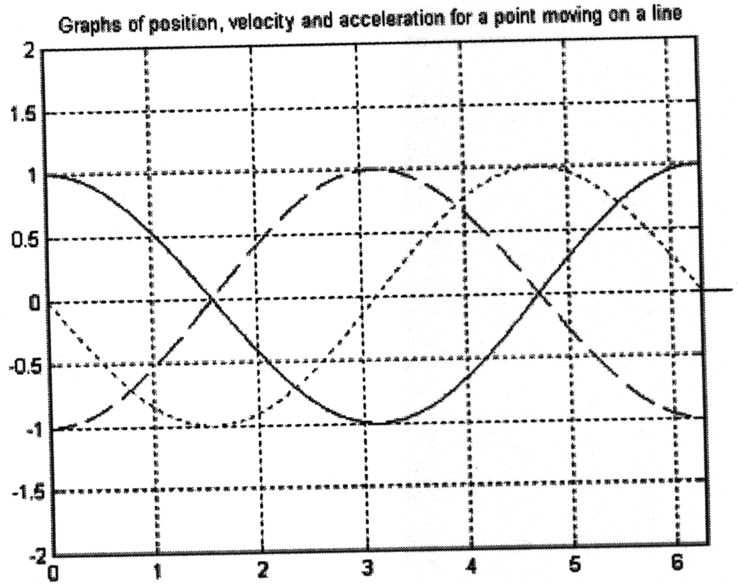
$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{a} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

and try to match up patterns.

$\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{a} = \lim\limits_{h \to 0} \frac{\cos(h+\pi)+1}{h}$ if we choose $f(x) = \cos x$ and $a = \pi$, so $\lim\limits_{h \to 0} \frac{\cos(h+\pi)+1}{h} = \cos'(\pi)$ (whatever the value of that is!) and

$\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim\limits_{x \to 1} \frac{x^4+x-2}{x-1}$ if we choose $f(x) = x^4 + x$ and $a = 1$. Then the limit gives $f'(1)$.

3. A point moves along a straight line. At time t its position is $s = f(t)$. The following figure contains the graphs of $f(t)$, $v(t)$ (the velocity function) and $a(t)$ (the acceleration function). Decide which is which.



Solution:

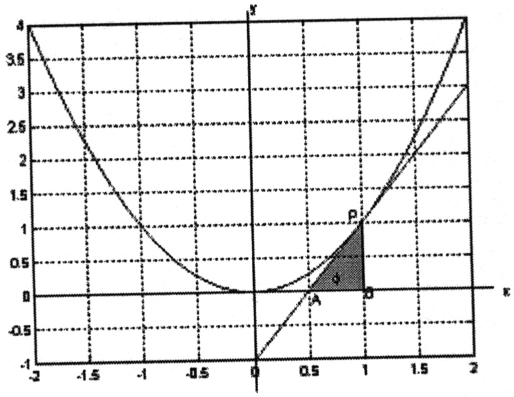
Call the solid graph f_1, the dotted graph f_2, and the dashed graph f_3.

If the dotted graph f_2 were the position, s: s would be decreasing from 0 to about $t = 1.5$, so v would be < 0 for those times; therefore v would have to be the dashed graph f_3; by elimination., this would mean $f_3{}' = \frac{dv}{dt} = a =$ the solid graph f_1 – which is impossible: v would be increasing from about 1.5 to about 3.1 but a would be negative there. So $s = f_2$ is impossible.

If the dashed graph f_3 were the position, s: s would be increasing from about $t = 0$ to about $t = 3.1$, so v would have to be > 0 for those times: neither f_1 nor f_2 could be v. So $s = f_3$ is impossible.

Therefore it must be that $s =$ the solid graph f_1. Then s is decreasing from $t = 0$ to about $t = 3.1$ so v must be negative for those times. Therefore $v = f_2$ and finally we conclude $a = f_3$.

4. Let $f(x) = x^2$. It is easy to work out from the definition of derivative that $f'(x) = 2x$. Let ℓ be the tangent line to the parabola $y = x^2$ at the point $(1, 1)$. The line ℓ make an angle ϕ where it crosses the positive x-axis (ϕ is called the "angle of inclination" for ℓ.) Find ϕ.



Looking at $\frac{\text{opposite}}{\text{adjacent}}$ in the shaded triangle, we see that $\tan\phi =$ slope of the tangent line at $(1, 1)$. This slope is $f'(1) = 2$. Therefore $\tan\phi = 2$, and $0 < \phi < \frac{\pi}{2}$, so $\phi = \arctan 2$. This is the exact answer. A calculator can then be used if you want an approximate value for this: $\arctan 2 \approx 1.1071$ (radians) which is about $63.4349°$.