**Math 131, Fall 2004**
**Discussion Section 7 Solutions**

1. Find the derivative

a) $f(x) = \frac{\tan x}{1+\sec x}$

Solution: Using the quotient rule: $f'(x) = \frac{(1+\sec x)(\sec^2 x) - (\tan x)(\sec x \tan x)}{(1+\sec x)^2}$

b) $\frac{dy}{dx}|_{x=\frac{\pi}{2}}$ if $y = x^2 + e^{\sin(\cos x)}$

Solution: The chain rule gives $\frac{dy}{dx} = 2x + e^{\sin(\cos x)} \frac{d}{dx}(\sin(\cos x))$
$= 2x + e^{\sin(\cos x)}(\cos(\cos x))\frac{d}{dx}(\cos x)$
$= 2x + e^{\sin(\cos x)}(\cos(\cos x))(-\sin x)$
$= 2x - (\sin x)e^{\sin(\cos x)}(\cos(\cos x))$
Therefore $\frac{dy}{dx}|_{x=\frac{\pi}{2}} = 2(\frac{\pi}{2}) - \sin(\frac{\pi}{2})e^{\sin(\cos(\frac{\pi}{2}))}(\cos(\cos(\frac{\pi}{2}))$
$= \pi - (1)e^0\cos(0)) = \pi - 1$

c) $g(z) = x^2 10^{\sin x}$

Solution: Using the product rule gives $g'(x) = (2x)(10^{\sin x}) + (x^2)\ln(10)10^{\sin x}(\cos x)$

2. Suppose $y = f(z) = z + \tan^3 z^5$ , where $z$ depends on $t$. Suppose $z$ is decreasing at a rate of 5 units/sec. Is $y$ increasing or decreasing? Why?

Solution: $y = f(z) = z + \tan^3 z^5 = z + (\tan z^5)^3$, so
$\frac{dy}{dz} = 1 + 3\,(\tan(z^5))^2 \frac{d}{dz}(\tan z^5)$
$= 1 + 3\,(\tan(z^5))^2 \sec^2(z^5)(5z^4) = 1 + 3\tan^2(z^5)\sec^2(z^5)\,(5z^4)$
$= 1 + 15z^4\tan^2(z^5)\sec^2(z^5)$

3. Suppose $F(x) = f(g(x))$. What is what is $F'(2)$ if:

| | |
|---|---|
| $g(2) = 6$ | $g'(2) = 4$ |
| $f'(2) = 1$ | $f'(6) = -2$ |

Solution: By the chain rule, $F'(x) = f'(g(x))g'(x)$, so
$F'(2) = f'(g(2)) \cdot g'(2) = f'(6) \cdot g'(2) = -8$

4. Let $f(x) = (2x+1)^3(4x-7)^9$. For what $x$'s is $f(x)$ increasing and for what $x$'$s$ is $f(x)$ decreasing ?

Solution: $f'(x) = 3(2x+1)^2(2)(4x-7)^9 + 9(4x-7)^8(4)(2x+1)^3$
$= (2x+1)^2(4x-7)^8[\,6(4x-7) + 36(2x+1)\,]$
$= (2x+1)^2(4x-7)^8(24x - 42 + 72x + 36)$
$= (2x+1)^2(4x-7)^8(96x-6) = 6(2x+1)^2(4x-7)^8(16x-1)$

Since $6(2x+1)^2(4x-7)^8 \geq 0$, the sign of $f'(x)$ matches the sign of $16x-1$ :

$16x - 1 > 0$ if $x > \frac{1}{16}$ and $16x-1 < 0$ if $x < \frac{1}{16}$. Therefore $f$ is increasing for $x > \frac{1}{16}$ and decreasing for $x < \frac{1}{16}$ (*so $f$ has a local minimum at* $\frac{1}{16}$).