Math 131, Fall 2004
Discussion Section 8 Solutions

In case it's needed, the formula (from the review sheet on logarithms) for converting logarithmic bases is:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

1. Find the derivative:

a) $y = \log_7 (x^2 + 1)$

<u>Solution</u>: The general formula (chain rule version) is: if $y = \log_a u$, then $\frac{dy}{dx} = \frac{1}{(\ln a)\, u}\frac{du}{dx}$.
Therefore $\frac{dy}{dx} = \frac{1}{(\ln 7)(x^2+1)}(2x) = \frac{2x}{(\ln 7)(x^2+1)}$

b) $y = \ln\left(\frac{(x^2+1)}{\sqrt[5]{\cos x}}\right)$

<u>Solution</u>: You can find $\frac{dy}{dx}$ by grinding away with the formula $\frac{dy}{dx} = \frac{1}{\frac{(x^2+1)}{\sqrt[5]{\cos x}}} \cdot \frac{d}{dx}\left(\frac{(x^2+1)}{\sqrt[5]{\cos x}}\right)$
$= \ldots$
but it's easier to simplify (using properties of logarithms) before differentiating:

$y = \ln\left(\frac{(x^2+1)}{\sqrt[5]{\cos x}}\right) = \ln(x^2+1) - \ln(\sqrt[5]{\cos x}) = \ln(x^2+1) - \ln((\cos x)^{1/5}) =$
$\ln(x^2+1) - \frac{1}{5}\ln(\cos x)$. Then
$\frac{dy}{dx} = \frac{1}{x^2+1}(2x) - \frac{1}{5}\frac{(-\sin x)}{\cos x} = \frac{2x}{x^2+1} + \frac{1}{5}\tan x$

c) $y = x^{(x^x)}$ (*To save some time you may use the fact, from class, that* $\frac{d}{dx}(x^x) = x^x(1 + \ln x)$)

<u>Solution</u>: Use logarithmic differentiation.
$\ln y = \ln(x^{(x^x)}) = x^x \ln x$. Then
$\frac{1}{y} \cdot \frac{dy}{dx} = x^x \frac{d}{dx}(\ln x) + (\ln x)\frac{d}{dx}(x^x) = x^x \cdot \frac{1}{x} + (\ln x)[x^x(1 + \ln x)]$
$= x^{x-1} + x^x(\ln x)(1 + \ln x)$, so
$\frac{dy}{dx} = y[x^{x-1} + x^x(\ln x)(1 + \ln x)] = x^{(x^x)}[x^{x-1} + x^x(\ln x)(1 + \ln x)]$
$= x^{(x^x)}x^{x-1}[1 + x(\ln x)(1 + \ln x)]$.

d) $y = \log_x 5 \quad (x > 1)$. Is this function always increasing? always decreasing ? neither?

Solution: 1) See the formula at the beginning for changing logarithmic bases. $y = \log_x 5 = \frac{\ln 5}{\ln x}$. Since $\ln x$ is increasing and $\ln x > 0$ for $x > 1$, $y = \frac{\ln 5}{\ln x}$ is decreasing.

2) Alternate solution: Since $y = \log_x 5 = \frac{\ln 5}{\ln x}$,

$\frac{dy}{dx} = -(\ln 5)(\ln x)^{-2} \cdot \frac{1}{x} = -\frac{\ln 5}{x(\ln x)^2}$. Since $\ln 5 > 0$ and $x > 1$, the derivative is always negative so y is always decreasing.

2. The volume of a sphere with radius r: $V(r) = \frac{4}{3}\pi r^3$

If the radius of a sphere is increased from 8 to 8.1 meters, then the exact change in volume $= \Delta V = V(8.1) - V(8) = \frac{4}{3}\pi(8.1)^3 - \frac{4}{3}\pi(8)^3$ (m^3)

We could actually work out the exact value ΔV. However, we might settle for an approximation in exchange for a little less computational work. We can use differentials:

$\Delta V \approx dV =$ ..... (m^3)

Solution: $\Delta V \approx dV = 4\pi r^2 dr$. When $r = 8$ and $dr = 0.1$, we get $\Delta V \approx dV = 4\pi(8)^2(.1) \approx 80.4248\,\text{m}^3$

(Just for comparison, the actual change is $\Delta V = \frac{4}{3}\pi(8.1)^3 - \frac{4}{3}\pi(8)^3 = 81.4343\,\text{m}^3$. The error in making this approximation is $\Delta V - dV \approx 81.4343 - 80.4248 = 1.0095$ an error of a little more than 1%.)